Chapter 3 PN Junction and Diode

3.2 PN Diode--Small Signal Model and Transient Response Model

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Outline

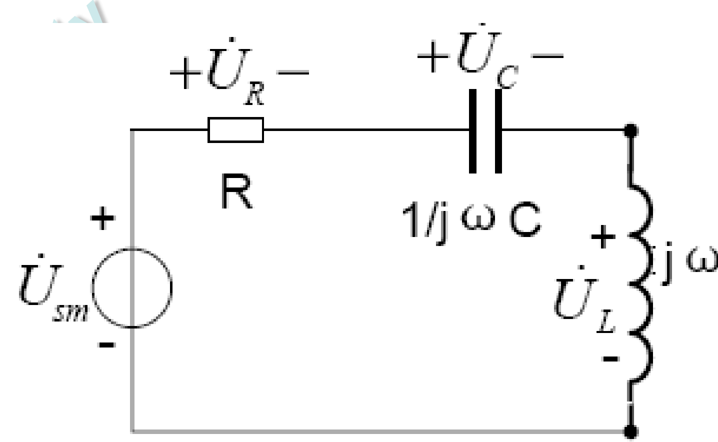
- ***Small Signal Model***
  - Introduction to Impedance and Admittance
  - Reverse-bias Capacitor
  - Forward-bias Diffusion Admittance
- ***Transient Response Model***
  - Turn-Off Transient of PN Diode
  - Turn-on Transient of PN Diode

Small Signal Model of PN Diode

Introduction to Impedance and Admittance

Impedance (阻抗,Z)

- **A sinusoidal(正弦） signal is applied on a device, the ratio between voltage phasor and current phasor.Z=U_m/I_m *(ohm)***
- $Z=Z_R+Z_c+Z_L$:
  - electrical resistance(电阻):Z_R= R
  - capacitor resistance(容抗):$Z_C=1/(j\omega C)$ =-j/(ωC (**ωis AC signal frequency, rad/s)**
  - Inductive resistance(感抗): $Z_L=j\omega L=j2\pi fL$ (f is AC single frequency, Hz)



- Z is a complex number: $Z=R+jX=|Z|\angle\theta_Z$
  - R=R_e(Z) is resistance component(阻抗的电阻分量)
  - X=I_m(Z) is reactance component (阻抗的电抗分量)
  - |Z| is the module of impedance(阻抗的模)
  - θ_Z is impedence argument (阻抗的幅角)

$$X_c=I_m(Z_c)=-\frac{1}{\omega C}\qquad\text{（容抗）}$$

$$X_L=I_m(Z_L)=\omega L\qquad\text{（感抗）}$$

Admittance(导纳)

- **Definition**
  - Admittance is the reciprocal（倒数） of impedance.
    - $Y=1/Z=I_m/U_m$ (unit, S=simense)
      - The admittance of electrical resistance (conductance) $Y_R=1/R=G$
      - The admittance of capacitor $Y_c=j\omega C$
      - The admittance of a inductor: $Y_L=1/j\omega L=-j\omega L$
    - Admittance is also a complex number:

$$|Y|=\frac{1}{|Z|}=\frac{I_m}{U_m}$$

$$Y=|Y|\angle\theta_y=G+jB$$

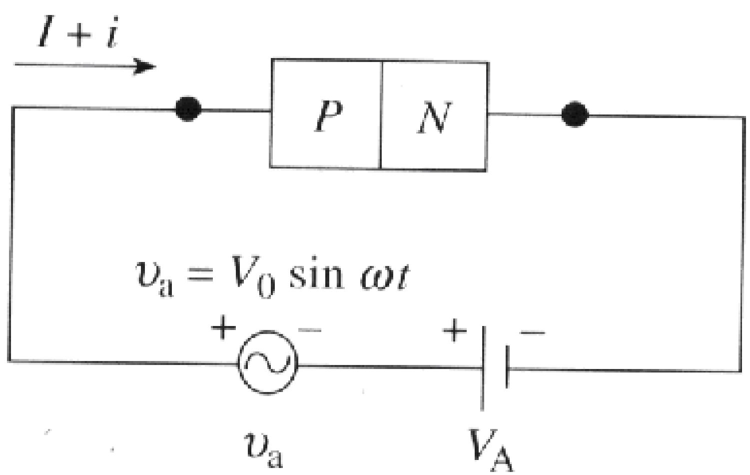
$$\theta_Y=-\theta_Z=\theta_i-\theta_u$$

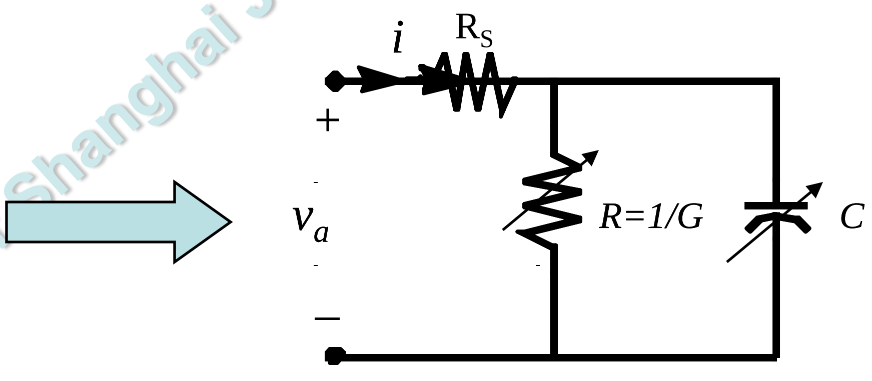
$B_c=I_m(Y_c)=\omega C$ 为电容的电纳，简称容纳。

$B_L=I_m(Y_L)=-\dfrac{1}{\omega L}$ 为电感的电纳，简称感纳。

Why Discuss Small Signal Model

- When semiconductor devices with pn junctions are used in linear amplifier circuits, the small-signal characteristics of the pn junction become important.
  - **for example, sinusoidal(正弦) signals are superimposed（施加） on the DC currents and voltages. V_A+v_a**





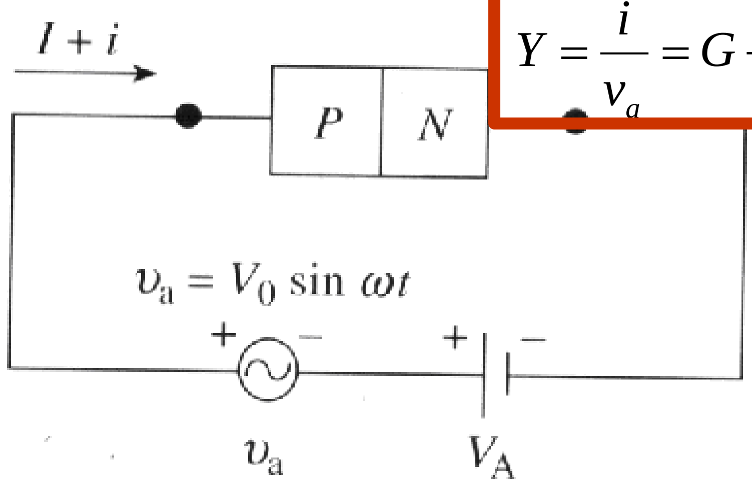
Small signal equivalent circuit

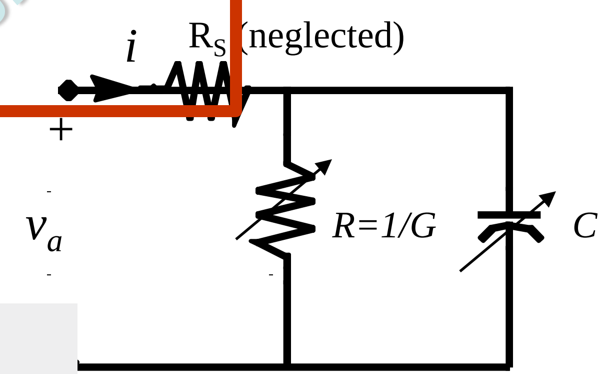
Small Signal Admittance (小信号导纳)

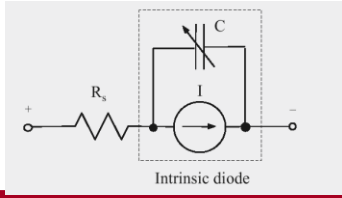
- Definition
  - **Small signal voltage/current:**
    - **When a small and low-frequency sinusoidal voltage superimposes on the DC bias**
  - **Small signal admittance (Y)**
    - **$Y=i/v_a=G+j\omega C$, where $j=\sqrt{-1}$, ω is the frequency of AC signal(rad/s) ,G is conductance**
    - **used to characterize the AC response of a passive device (无源器件),e.g., diode.**
    - **Reverse –bias: $G\rightarrow 0$, $Y\approx j\omega C$**
    - **Forward-bias: G cannot be neglected, and need consider minority's contribution.**
      - **C is very small**

Small Signal Admittance of PN Diode

$$Y=\frac{i}{v_a}=G+j\omega C$$

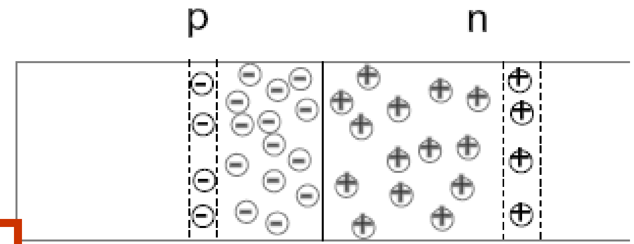






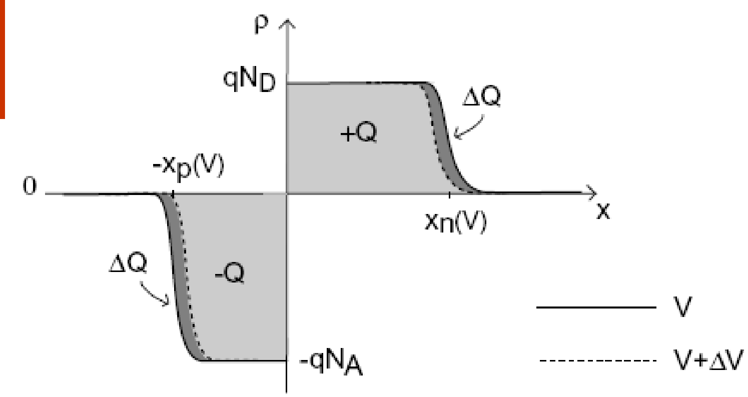
Reverse-bias Capacitor

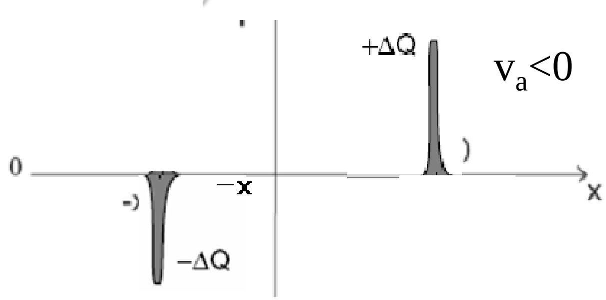
Depletion Capacitor

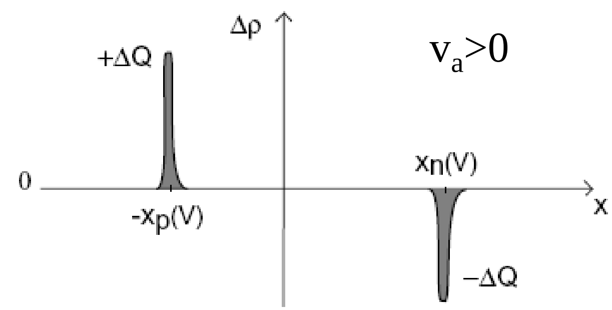


- **When a diode is reverse biased, it equals to a capacitor, which varies from the width of depletion layer.**
- **Quasi static change:**
  - If majority carrier, which can quickly flow into or out of depletion layer boundary, can response to the small AC signal v_a
  - Majority carrier in Si:<10^{-10}s, has high frequency response ability.

$$\boxed{C_J = \frac{K_s \varepsilon_0 A}{W}}$$







Depletion Capacitor

$$W \cong \left[\frac{2\varepsilon_s}{qN_B}(V_{bi}-V_A)\right]^{1/2}$$ For a single-side abrupt junction

$$W \cong \left[\frac{12\varepsilon_s}{qa}(V_{bi}-V_A)\right]^{1/3}$$ For a linear graded pn junction.

where N_B - the impurity concentration of the light doped side.

$N_B(x) = bx^m$

where b > 0 and m is a constant.

$$\begin{cases} m = 1 \Rightarrow \text{single-side linear graded junction} \\ m = 0 \Rightarrow \text{single-side abrupt junction} \\ m < 0 \Rightarrow \text{(single-side) hyperabrupt junction} \\ \qquad \text{(by implantation and epitaxy} \end{cases}$$

for a single-side exponential distribution pn diode,

$$\begin{cases} m = 0, b = N_B \Rightarrow \text{single-side abrupt junction} \\ m = 1, b = a/4 \Rightarrow \text{single-side linear junction} \end{cases}$$

$$\boxed{\therefore C_J = \frac{K_S\varepsilon_0 A}{\left[\frac{(m+2)K_S\varepsilon_0}{qb}(V_{bi}-V_A)\right]^{1/(m+2)}}}$$

$$\because C_{J0} = C_J\big|_{V_A=0} = \frac{K_S\varepsilon_0 A}{\left[\frac{(m+2)K_S\varepsilon_0}{qb}V_{bi}\right]^{1/(m+2)}}$$

$$\boxed{\therefore C_J = \frac{C_{J0}}{\left[1-\frac{V_A}{V_{bi}}\right]^{1/(m+2)}}}$$

Application of Reverse-Bias Capacitor (I) Varactor (变容二极管)

- **Definition**
  - Varactor=variable+reactor(电抗器)
    - reactance of a capacitor=1/**jωC**
  - Capacitor varies from voltage to voltage
- **Modulation ratio (TR)**

$$TR \equiv \frac{C_J(V_{A1})}{C_J(V_{A2})} \cong \left(\frac{V_{A1}}{V_{A2}}\right)^{1/(m+2)} \quad (\text{reverse bias, } V_A >> V_{bi})$$

  - The largest capacitor ratio among a voltage range
  - The smaller the m, the larger the TR➔m=-1 (single-side hyperabrupt junction) can get the largest TR.
- **Application (m=0,m=-1)**
  - Parameter magnification, resonant wave generation, mixing frequency, demodulation, voltage-variable modulation

Application of Reverse-Bias Capacitor (II)

Parameters' Extraction and Measuremnt of Dopant Distribution

- **Reverse-bias C-V measurement to determine some device parameters in light doped side**
  - Doped concentration: n_B
  - Dopant distribution

$$\text{if } m=0,\ b=N_B \Rightarrow \text{single-side abrupt junction}$$

$$C_J = \frac{K_S\varepsilon_0 A}{\left[\frac{(m+2)K_S\varepsilon_0}{qb}(V_{bi}-V_A)\right]^{1/(m+1)}} = \frac{K_S\varepsilon_0 A}{\left[\frac{2K_S\varepsilon_0}{qN_B}(V_{bi}-V_A)\right]^{1/2}}$$

then:

$$\boxed{\frac{1}{C_J^2} = \frac{2}{qN_B K_S\varepsilon_0 A^2}(V_{bi}-V_A)}$$

$\Rightarrow$ the curve of $\frac{1}{C_J^2} \sim V_A$ is a line,

the reciprocal of the slop is proportional to N_B

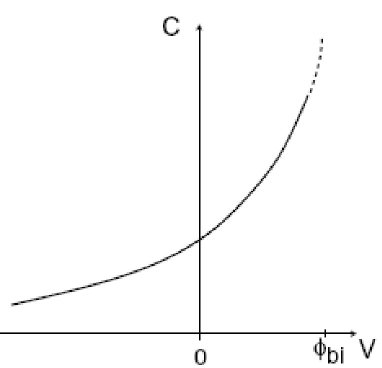
the intercept equals to V_{bi} (not accurate)

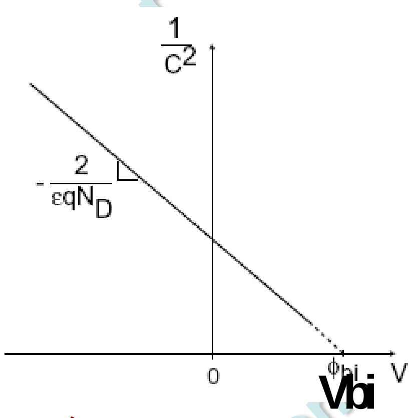
$$\boxed{N_B(x) = \frac{2}{qK_S\varepsilon_0 A^2\left|\frac{d(1/C_J^2)}{dV_A}\right|},}$$

where x is the distance from the location in light-doped side to the metallurgical junction

$\because$ for single side heavy doped pn junction: $W \approx x$

$$\boxed{\therefore x = \frac{K_S\varepsilon_0 A}{C_J}}$$





Example

If the slope of the $(1/C_{dep})^2$ *vs.* V_A characteristic is -2x10^{23} F^{-2} V^{-1}, the intercept is 0.84V, and *A* is 1 μm^2, find the lighter and heavier doping concentrations N_l and N_h.

Solution:

$$N_l = 2/(slope \times q\varepsilon_s A^2)$$

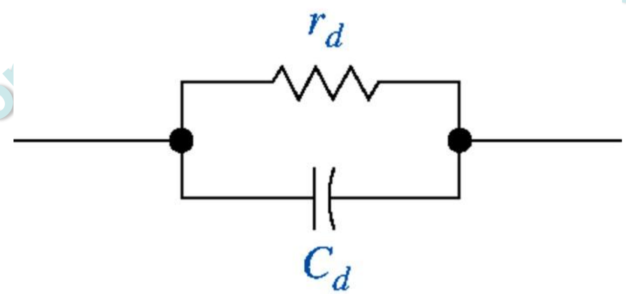
$$= 2/(2\times10^{23}\times1.6\times10^{-19}\times12\times8.85\times10^{-14}\times10^{-8}\,\mathrm{cm}^2)$$

$$= 6\times10^{15}\ \mathrm{cm}^{-3}$$

$$V_{bi} = \frac{kT}{q}\ln\frac{N_h N_l}{n_i^{\,2}} \Rightarrow N_h = \frac{n_i^{\,2}}{N_l}e^{\frac{qV_{bi}}{kT}} = \frac{10^{20}}{6\times10^{15}}e^{\frac{0.84}{0.026}} = 1.8\times10^{18}\ \mathrm{cm}^{-3}$$

Reverse-bias Conductance

- **All standard capacitor have their conductance**
- **For a reverse-bias PN diode, there is a very small conductance**



(a)

Small signal equivalent circuit for a reverse-biased PN diode

$$\text{low frequency conductance}: G_0 = \frac{dI}{dV_A}$$

$$\because I = I_0\left(e^{qV_A/kT} - 1\right)$$

$$\therefore G_0 = \frac{dI}{dV_A} = \frac{q}{kT} I_0 e^{qV_A/kT} = \frac{q}{kT}(I + I_0)$$

$$\text{if } \mathrm{V_A} >> \frac{q}{kT} = V_{th},\quad \mathrm{I} \to -I_0,\ G_0 \to 0$$

$$\text{if } \mathrm{V_A} \sim n * \frac{q}{kT} = nV_{th}\,(n < 10),$$

DC thermal G-R current is dominant

$$\mathrm{I_{G\text{-}R}} = -\frac{qAn_i}{2\tau_0} W$$

$\therefore$ parasitical conductance

$$G_0 = \frac{dI_{G-R}}{dV_A} = \frac{qAn_iW/2\tau_0}{(m+2)(V_{bi} - V_A)}$$

$$r_d = \frac{1}{G_0}$$

Forward-bias Diffusion Admittance

Capacitors of a Forward-biased PN Diode

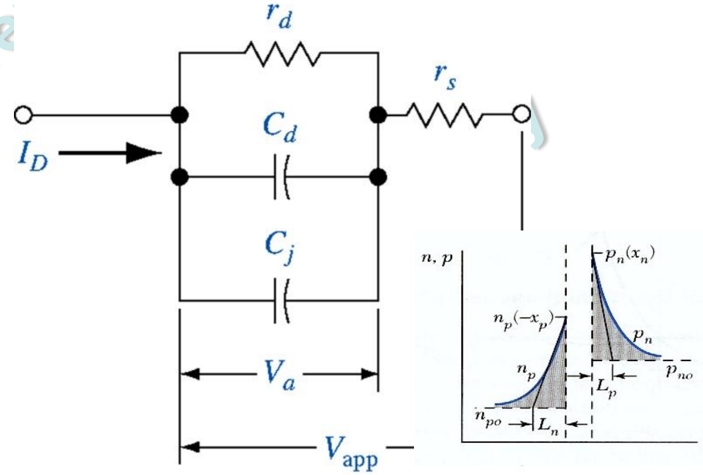
Two types of capacitance associated with a pn junction forward-biased:

- C_J : Junction Capacitor (*depletion capacitance)* due to variation of depletion charge
  - dominates at low forward biases, reverse biases
- C_D :diffusion capacitance due to variation of stored minority charge in the quasi-neutral regions'
  - dominates at moderate to high forward biases
  - **Caused by excess minority carrier charge Q_n and Q_p in neutral regions.**
  - **Only important in forward bias.**
  - **For a one-sided p+n junction, $Q_P >> Q_N$ ➔ $Q = Q_P + Q_N \cong Q_P$**

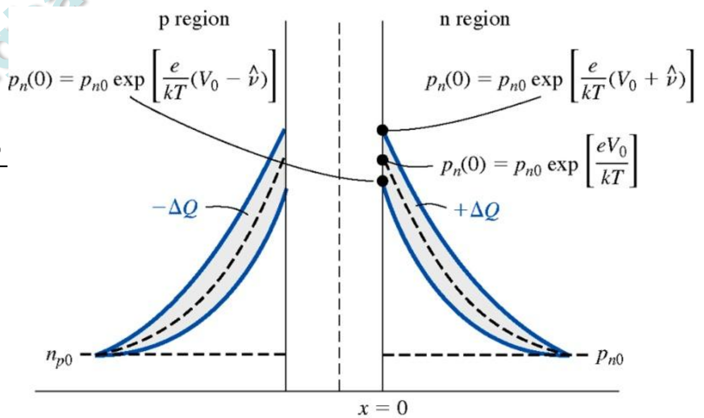
$$C_D = \frac{dQ_p}{dV} = \frac{q}{kT} qAL_p p_{n0} \exp\left(\frac{qV_A}{kT}\right) = \frac{q}{kT} I\tau_p = \frac{I\tau_p}{kT/q} = \frac{I\tau_p}{V_{th}}$$

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon_s}{q}(V_{bi} - V_A)\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

Small signal equivalent circuit for a forward-biased PN diode



(b)



Diffusion Admittance(扩散导纳)

- Diffusion admittance
  - **Minority carriers accumulate on the boundary of depletion layer due to diffusion current➔a admittance is produced due to minority carriers' charge caused by small signal.➔diffusion admittance G_D**
  - **Diffusion resistance: $r_d = 1/G_d$**

$$Y = G_D + j\omega(C_D + C_J)$$

$$\approx G_D + j\omega C_D = Y_D$$

$$\because I = I_0\left(e^{qV_A/kT} - 1\right)$$

$$\therefore Y = \frac{dI}{dV_A} = \frac{q}{kT} I_0 e^{qV_A/kT} = \frac{q}{kT}\left(I + I_0\right)$$

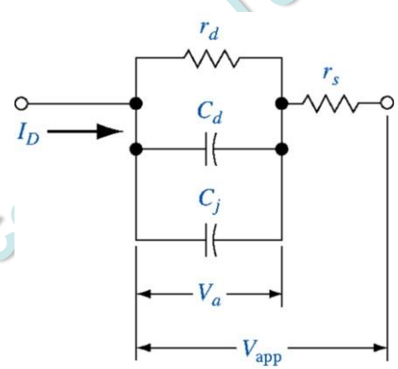
large forward - bias, $I >> I_0$

$$\therefore Y_D = \frac{q}{kT}\left(I + I_0\right) \approx \frac{q}{kT} I = \frac{I}{V_{th}}$$

Diffusion resistance:

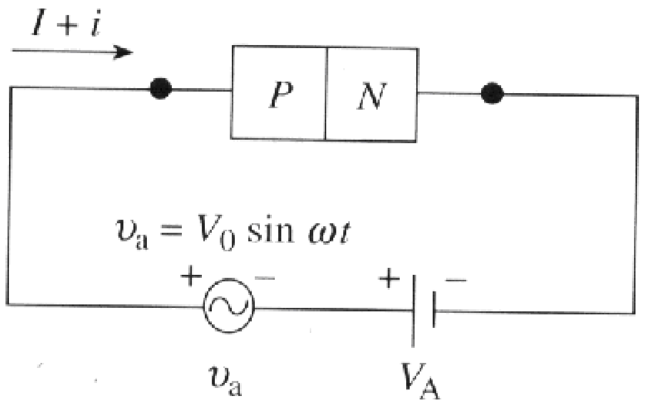
$$r_d = \frac{1}{Y_D} \approx \frac{V_{th}}{I}$$

Small signal equivalent circuit for a forward-biased PN diode



(b)

Summary: Small Signal Model for Forward Bias

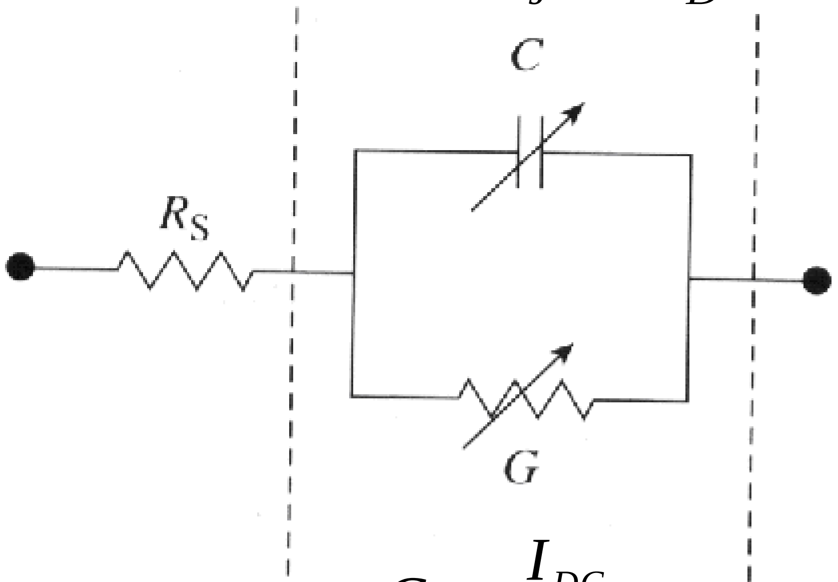


$$I_{DC} = I_0(e^{qV_A/kT} - 1)$$

Depletion capacitance $C_J = \dfrac{A\varepsilon_s}{W}$

Diffusion capacitance $C_D = \dfrac{I_{DC}\tau_p}{kT/q}$

$$C = C_J + C_D$$



Conductance $G \cong \dfrac{I_{DC}}{kT/q}$

- **For a reverse biased PN junction, it equals to a capacitor whose capacitance can reduce with the increase of reverse voltage.**
  - reverse-biased junction capacitor and varactor ;
  - C-V measurement: determine the average doping concentration or profile on light doped side.
- **For a forward-biased PN, minority carriers accumulate in the quasi-neutral zone which is very nearby depletion zone.**

Transient Response Model

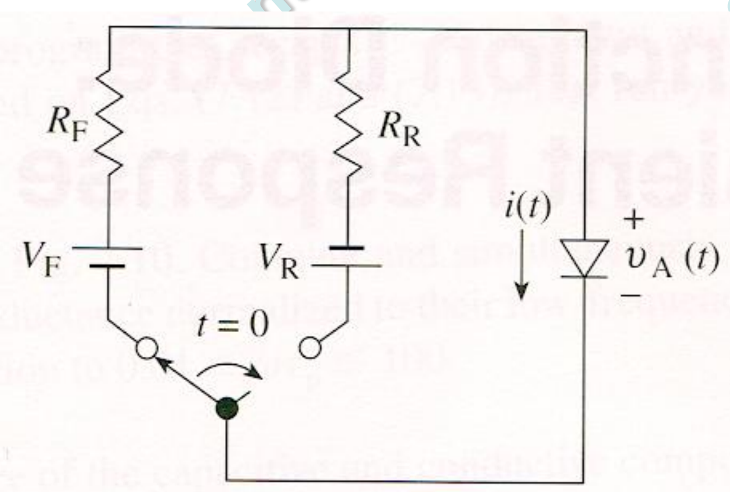
Contents on Semiconductor Device Analysis

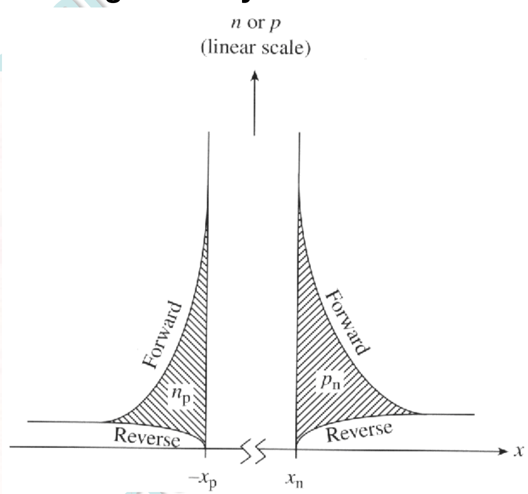
- Internal electrostatic model (equilibrium)
- Steady state response model (voltage stress, DC V_A)
- Small signal response model (V_A + v_a)
- Transient response Model (On/Off)

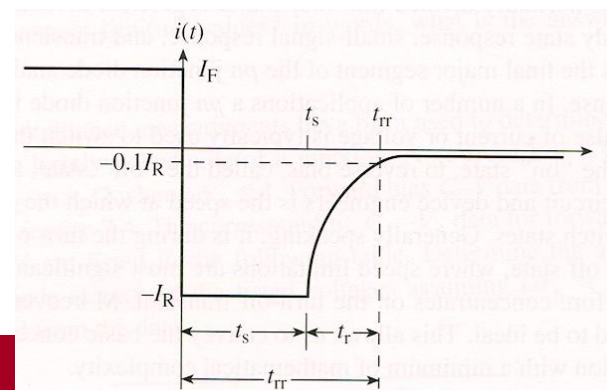
Turn-Off Transient of PN Diode

Turn-Off Transient of PN Diode

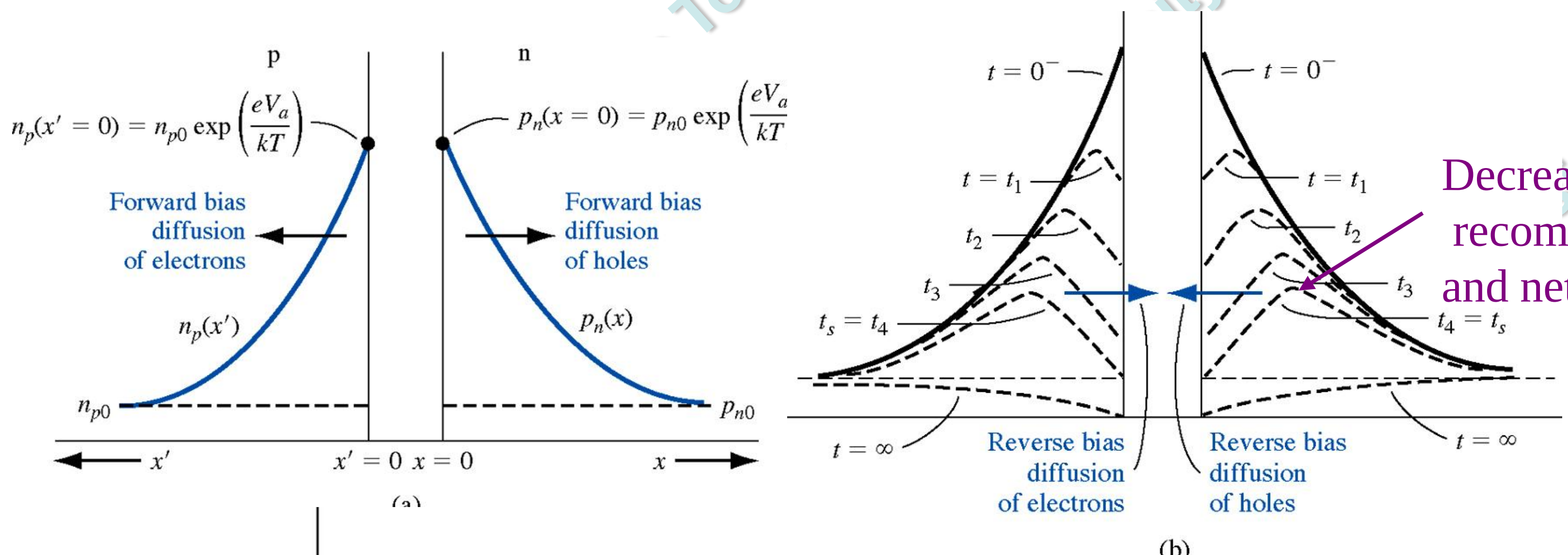
- Suppose a pn-diode is forward biased, then suddenly turned off at time $t = 0$. Because of C_D, the voltage across the pn junction depletion region cannot be changed instantaneously.
- The delay in switching between the ON and OFF states is due to the time required to change the amount of excess minority carriers stored in the quasi-neutral regions.
- In order to turn the diode off, the excess minority carriers must be removed by net carrier flow out of the quasi-neutral regions and/or recombination
  - Carrier flow is limited by the switching circuitry

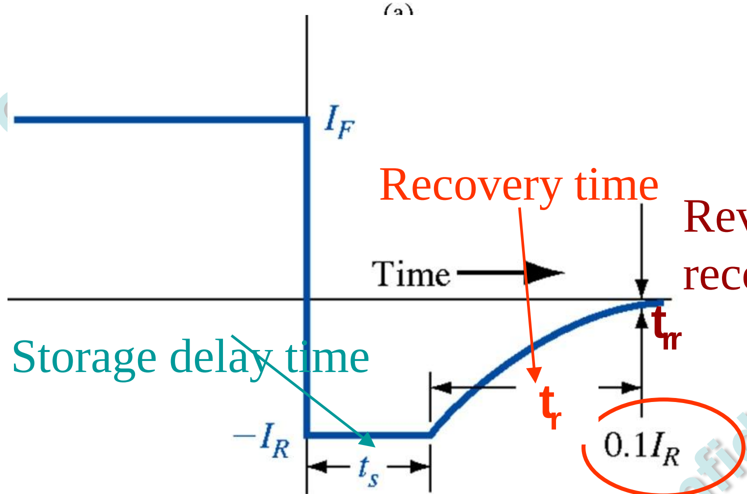


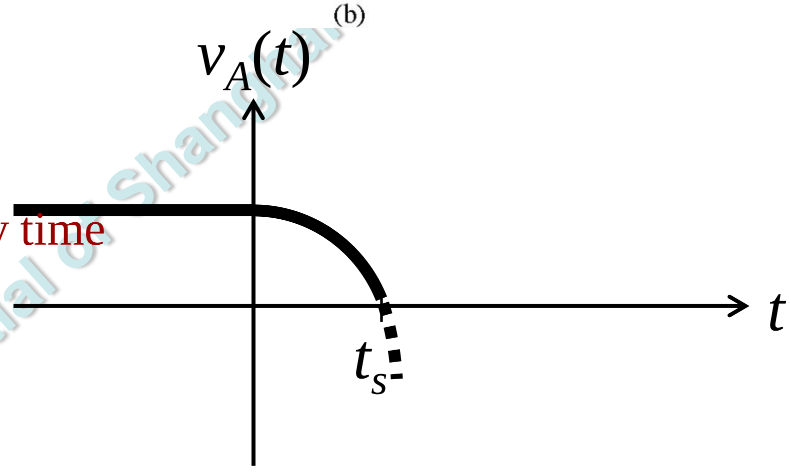




Turn-Off Transient of PN Diode







Excess Minority Removal Mechanism

- **Two mechanisms to remove excess minority in quasi-neutral zone (e.g., p^+n)**
  - Recombination: limited by the minority lifetime (τ_p)
  - Net carrier drift to return the other side due to the built-in field and reverse-bias voltage.
- **t_s is the primary "figure of merit" used to characterize the transient response of pn junction diodes**

Assumptions:

1) Comparing with power voltage (V_F and V_{Rd}), the largest forward voltage drop (V_{ON}) is small enough

$$I_F=\frac{V_F-V_{ON}}{R_F}\cong\frac{V_F}{R_F}\qquad I_R=\frac{V_R+v_A|_{0<t\le t_s}}{R_R}\cong\frac{V_R}{R_F}$$

For p^+n diode forward-biased,

according to Continuous Equation of Excess Hole charge

$$\frac{dQ_P}{dt}=I_{DIFF}-\frac{Q_P}{\tau_p}$$

when $0^+\le t\le t_s$, $I_{DIFF}=-I_R=$ cosntant

$$\frac{dQ_P}{dt}=-\left(I_R+\frac{Q_P}{\tau_p}\right)\qquad 0^+\le t\le t_s$$

$$\int_{Q_P(0^+)}^{Q_P(t_s)}\frac{dQ_P}{I_R+Q_P/\tau_p}=-\int_{0^+}^{t_s}dt=-t_s$$

$$\therefore t_s=-\tau_p\ln\left(I_R+Q_P/\tau_p\right)\Big|_{Q_P(0^+)}^{Q_P(t_s)}=\tau_p\ln\left[\frac{I_R+Q_P(0^+)/\tau_p}{I_R+Q_P(t_s)/\tau_p}\right]$$

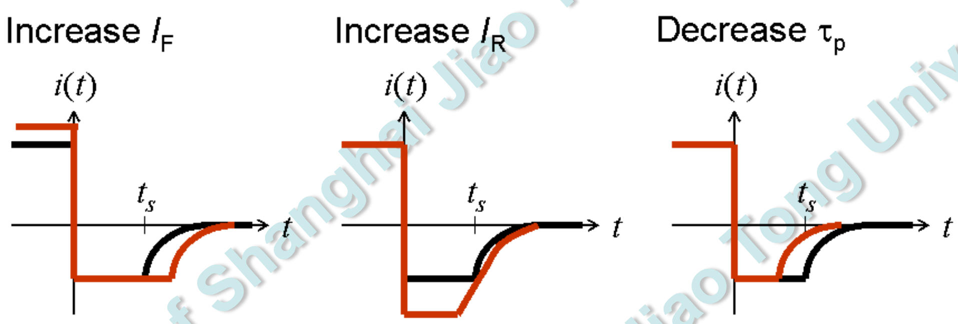
before turning off it is a steady-state:

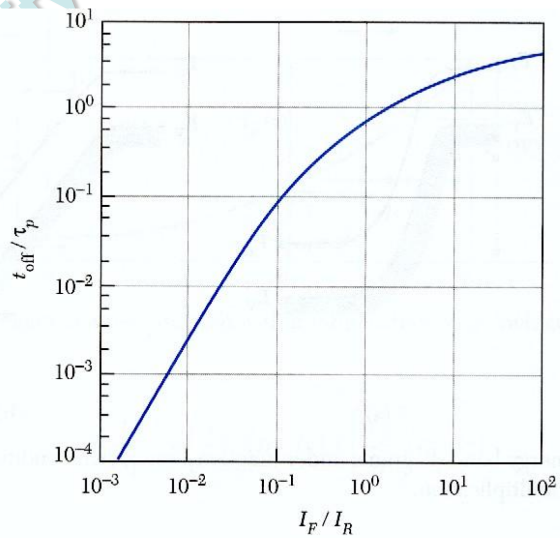
$$\frac{dQ_P}{dt}=i_{DIFF}-\frac{Q_P}{\tau_p}=0\Rightarrow i_{DIFF}=\frac{Q_P(0^-)}{\tau_p}=\frac{Q_P(0^+)}{\tau_p}=I_F$$

when assume $Q_P(t_s)=0$ at $t=t_s$

$$\boxed{\therefore t_s=\tau_p\ln\left[\frac{I_R+Q_P(0^+)/\tau_p}{I_R+Q_P(t_s)/\tau_p}\right]=\tau_p\ln\frac{I_R+I_F}{I_R}=\tau_p\ln(1+\frac{I_F}{I_R})}$$

Examples (qualitative)





$$t_s = \tau_p \ln\left[\frac{I_R + Q_P(0^+)/\tau_p}{I_R + Q_P(t_s)/\tau_p}\right] = \tau_p \ln\frac{I_R + I_F}{I_R} = \tau_p \ln\left(1 + \frac{I_F}{I_R}\right)$$

- **Larger N_T (trap center concentration),e.g, doping Au in Si➔Smaller minority lifetime (τ_p or τ_n)➔ smaller t_s**
  - Larger N_T ➔increase I_{G-R} (I_0)➔for sub ns on/off speed field: BJT or MOS
- **smaller I_F /I_R ➔ smaller t_s**
- **Stepped Recovery Diode with extreme steep junction: p–i–n diode (t_s ~1us, t_r~1ns)**
  - A narrow and light doped semi insert between high doped semi
  - Application:Pulse generator.

Turn-On Transient of PN Diode

- Consider a **p⁺n diode** ($Q_p >> Q_n$):
  - **Circuit forces a positive current I_F to flow through the diode beginning at time t=0**

According to Charge Control Model:

$$\frac{dQ_p}{dt} = i - \frac{Q_p}{\tau_p} = I_F - \frac{Q_p}{\tau_p} \quad \text{for } t \geq 0^+$$

$$Q_p(t) = I_F \tau_p \left(1 - e^{-t/\tau_p}\right)$$

If we assume that the build-up of stored charge occurs quasi-statically so that

$$Q_p(t) = I_{diffusion}\tau_p = I_0\left(e^{qv_A/kT} - 1\right)\tau_p$$

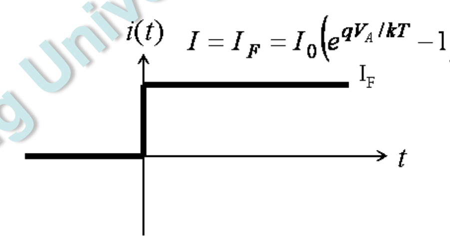
$$\therefore v_A(t) = \frac{kT}{q}\ln\left[1 + \frac{I_F}{I_0}\left(1 - e^{-t/\tau_p}\right)\right]$$

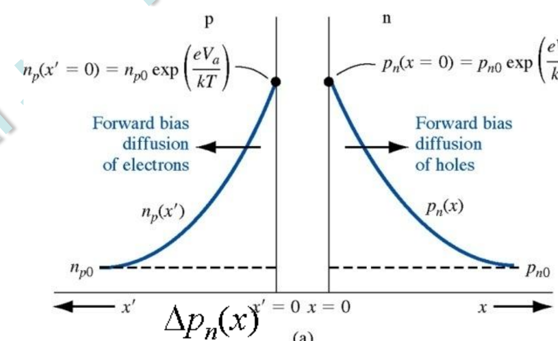
- If t_p is large(means that there is no recombination) then the time required to turn on the diode is approximately $\Delta Q/I_F$

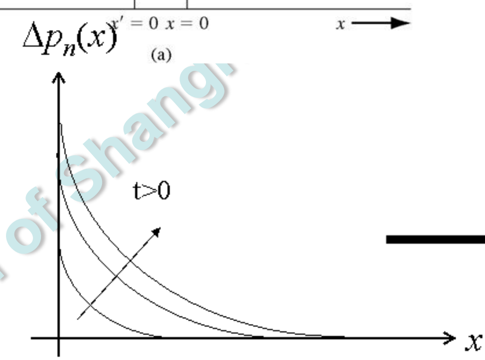
where $\Delta Q = \Delta Q_p + \Delta Q_j$

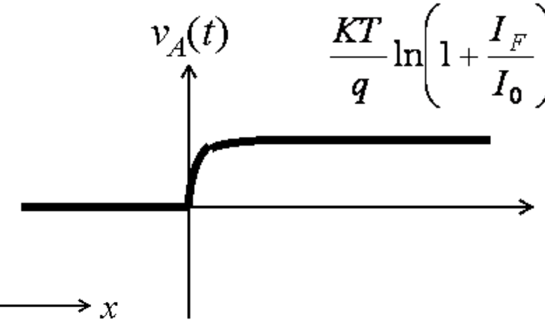
ΔQ_p - minority charges stored in n-Si

ΔQ_j – charges used to form junction depletion layer.









Summary

- When a pn junction is switched from forward bias to reverse bias, the stored excess minority carrier charge must be removed from the junction.
- The time required to remove this charge is called the storage time and is a limiting factor in the switching speed of a diode.

Homework

- **P230:**
  - 7.1 (a, b, c,d f,g,h)
  - *7.6(Matlab)
  - 7.9
- **P245**
  - 8.1(a,b,d,e)
  - 8.2
  - 8.5