Lecture 7.
Element Stamping

Guoyong Shi, PhD
shiguoyong@ic.sjtu.edu.cn
School of Microelectronics
Shanghai Jiao Tong University
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Outline

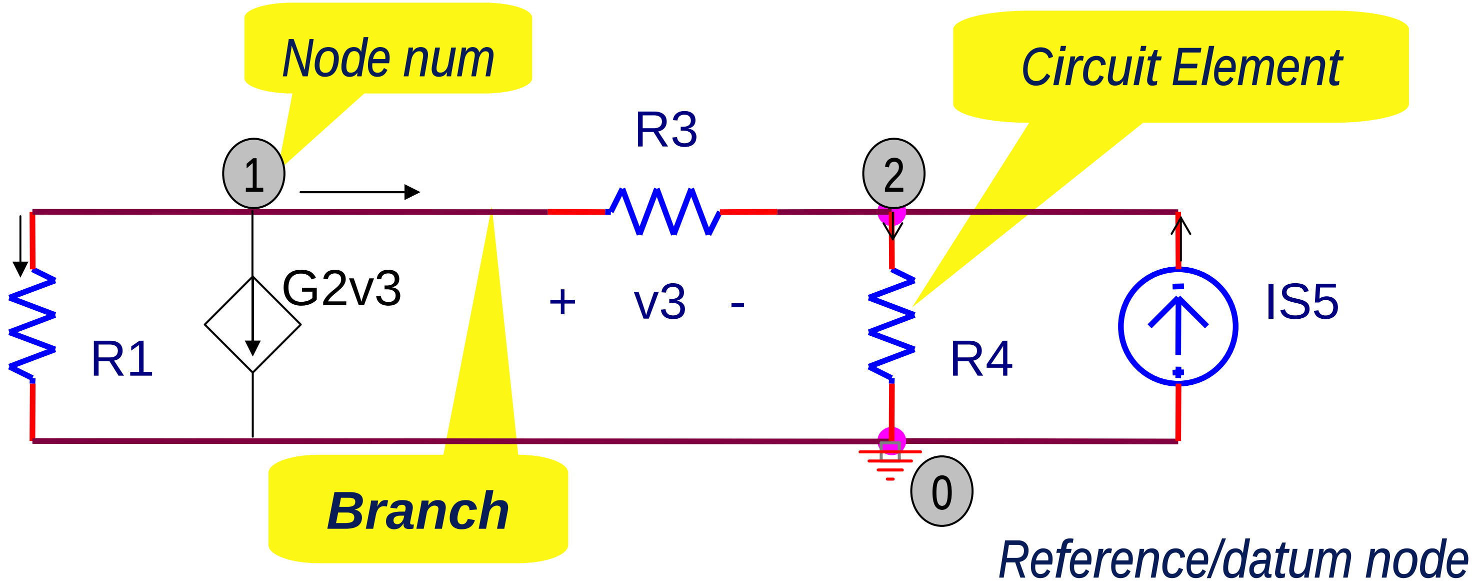
- **Basic Concepts**
  - **KVL/KCL**
  - **Circuit Element Equations**
- **Sparse Tabular Analysis (STA)**
- **Nodal Analysis**
- **Modified Nodal Analysis (MNA)**

- **Part 1: Static Element Stamping**

Formulation of Circuit Equations

- **Kirchoff Current Law (KCL)**
- **Kirchoff Voltage Law (KVL)**
- **Circuit Element Equations**

Basic Concepts



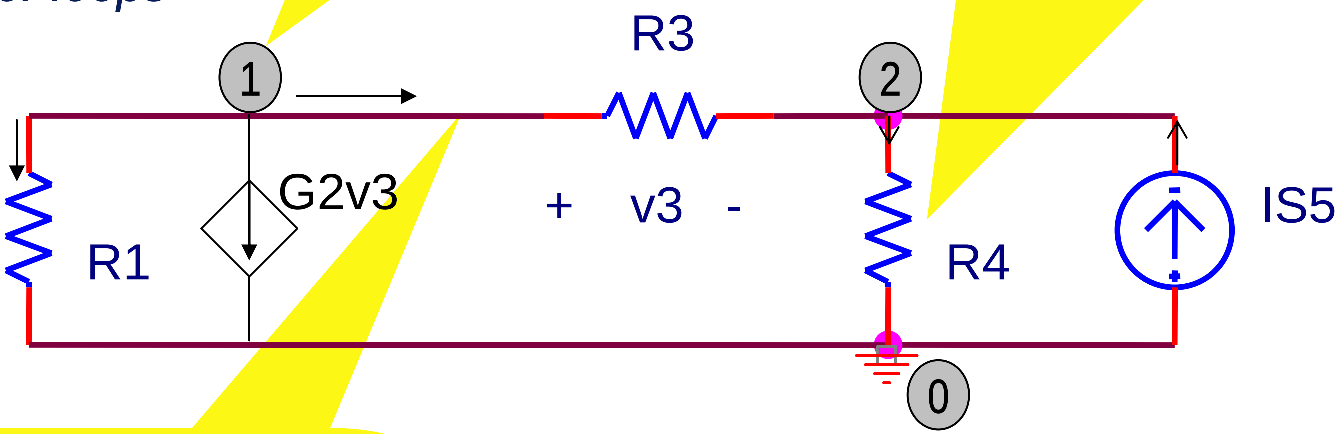
Basic Physical Quantities

KCL for nodes

KVL for loops

Node/Terminal Voltage e

Every circuit element (1 or 2 ports) is characterized by (i,v) equations



Branch current i

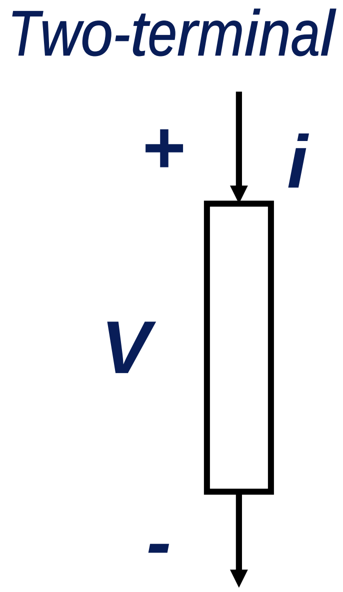
Branch voltage v

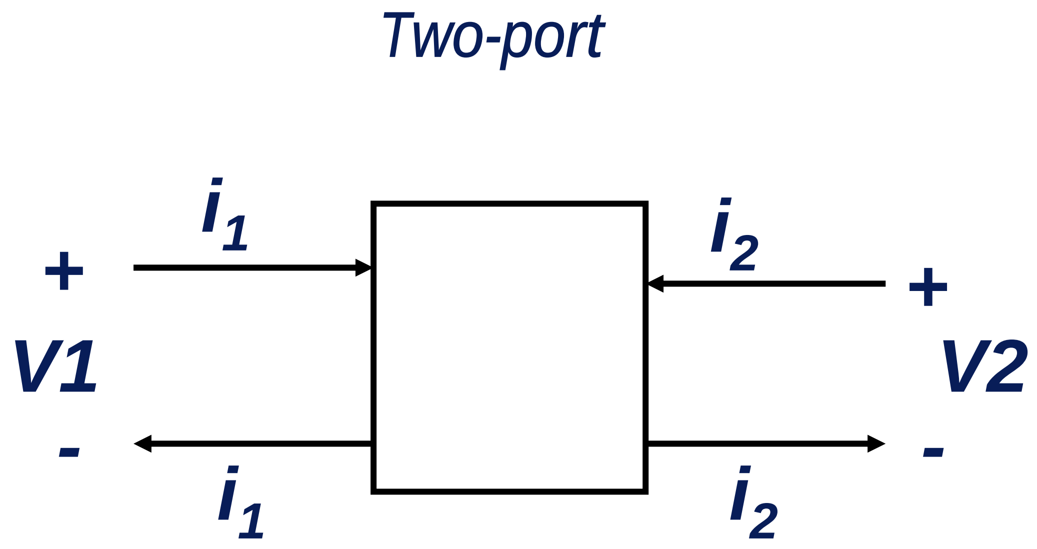
Reference node: e = 0

Circuit Element Equations

- Mathematical models of circuit components are expressed in terms of ideal elements:
  - Inductors
  - Capacitors
  - Resistors
  - Current Sources
  - Voltage Sources
  - Two Ports
  - ……….
- Physical quantities – current, voltage
- Some times, we need to use quantities: charge (nonlinear capacitor), flux (nonlinear inductor)

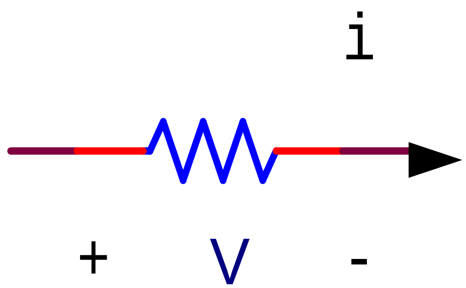
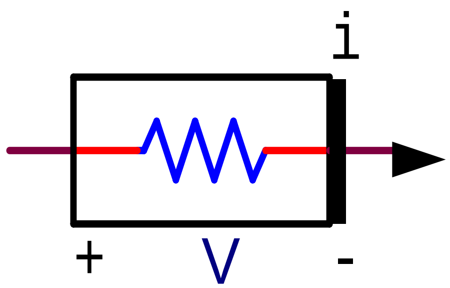
Reference Directions



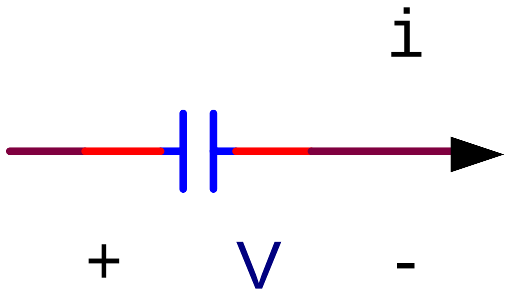
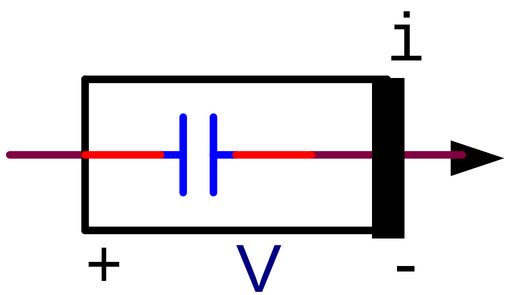


- *i and v are branch currents and voltages, respectively*
- *(Default) For each branch, current is directed from higher potential to lower potential*

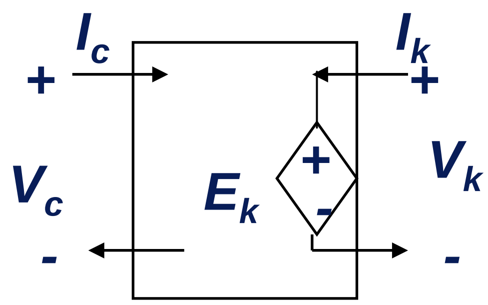
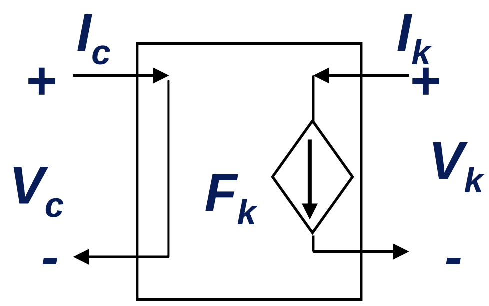
Resistor

Resistors	Symbol	Voltage controlled	Current controlled
Linear		**i = (1/R) v**	**v = R i**
Nonlinear		**i = i (v)**	**v = v (i)**

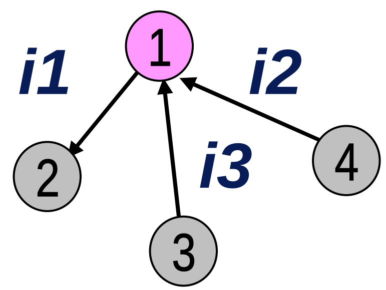
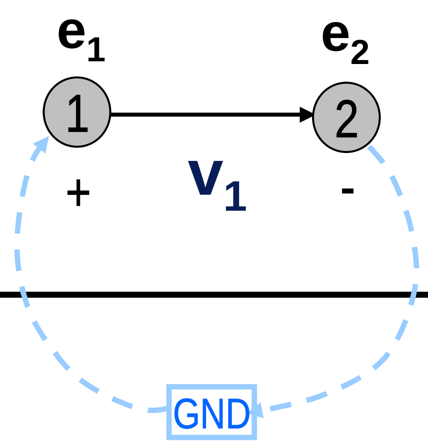
Capacitor

Capacitor	Symbol	Voltage controlled
Linear		**q = C v** **i = dq / dt** **Time-invariant C:** **i = C dv/dt**
Nonlinear		**q = q (v)** **i = dq / dt** **Time-invariant C:** **i = C(v) dv/dt**

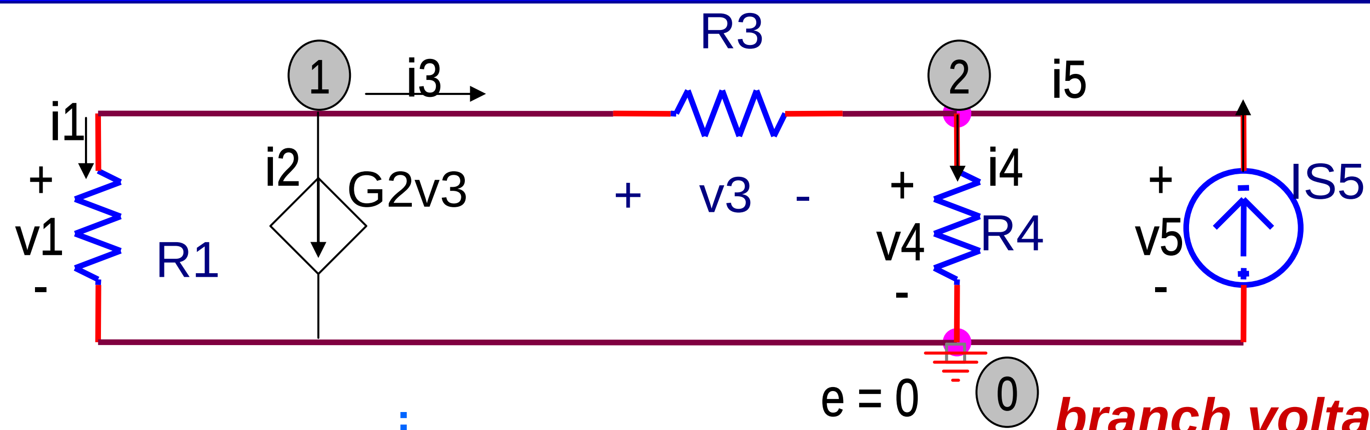
Two-Port Elements

Controlled Sources	Symbol	linear	Nonlinear
VCVS		$v_k = E_k\, v_c$ $i_c = 0$	$v_k = v_k(v_c)$ $i_c = 0$
CCCS		$i_k = F_k\, i_c$ $v_c = 0$	$i_k = i_k\,(i_c)$ $v_c = 0$

Topological Equations

KCL **(branch currents)** **Current leaving a node is "+"**		1 $i1\text{-}i2\text{-}i3 = 0$
KVL **(nodal voltages)** **Voltage dropping is "+"**		$v_1 + e_2 - e_1 = 0$

Matrix Forms



Nodal voltages

branch voltage vector

A

i

$$\begin{array}{c} 1 \\ 2 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i1 \\ i2 \\ i3 \\ i4 \\ i5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} e1 \\ e2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

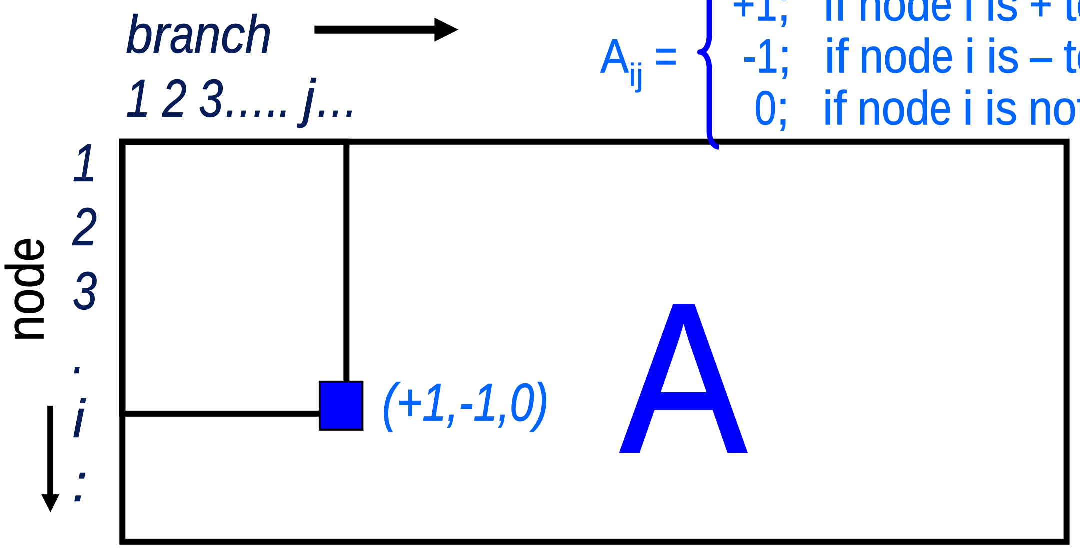
KCL: $A\,i = 0$

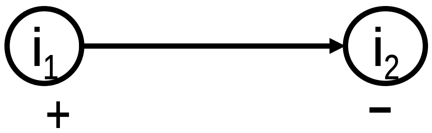
KVL $\quad v - A^T e = 0$

Tellegen's equation $i^T v = 0$
(conservation of energy)

Incidence Matrix A

$$A_{ij} = \begin{cases} +1; & \text{if node i is + terminal of branch j} \\ -1; & \text{if node i is – terminal of branch j} \\ 0; & \text{if node i is not connected to the branch j} \end{cases}$$





Each branch is directed

Properties

- **A is unimodular (*all minors equal to 1, -1, or 0*)**
- **Only 2 nonzero entries in any column**
- **Sum of all rows of A is a zero vector.**

  Thus, pick a node as the reference (ground) node

Equation Assembly

- **How does a computer assemble equations from the circuit description (netlist)?**
- **Two systematic methods:**

1. Sparse Tableau Analysis (STA)

Used by early ASTAP simulator (IBM)

2. Modified Nodal Analysis (MNA)

Used by SPICE simulators

Sparse Tableau Analysis (STA)

Proposed by (Brayton, Gustavson, Hachtel 1969-71)

- **Write KCL : $Ai = 0$** n equations (one for each node)
- **Write KVL : $v - A^T e = 0$** b equations (one for each branch)
- **Write Circuit Element (Branch) Equations :**

$$K_i i + K_v v = S$$

b equations

current controlled

voltage controlled

sources

Sparse Tableau Analysis

Put all (n + 2b) equations together:

$$\begin{bmatrix} A & 0 & 0 \\ 0 & I & -A^T \\ K_i & K_v & 0 \end{bmatrix} \begin{bmatrix} i \\ v \\ e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ S \end{bmatrix}$$

sparse tableau

n + 2b unknowns

n = #nodes

b = #branches

Advantages of STA

- **STA can be applied to any (linearized) circuit**
- **STA equations can be assembled directly from netlist**
- **STA coefficient matrix is very sparse**

$$\begin{pmatrix} \overset{2b}{A} & 0 & 0 \\ 0 & \overset{b}{I} & \overset{2b}{-A^T} \\ \underset{b}{K_i} & \underset{b}{K_v} & 0 \end{pmatrix} \begin{pmatrix} i \\ v \\ e \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ S \end{pmatrix}$$

(2b+2b+b+b+b) nonzeros

$\therefore$ ***sparsity is*** $\dfrac{7b}{(n+2b)^2}$

<u>Caution:</u>

Sophisticated programming techniques and data structures are required for achieving the time and memory efficiency

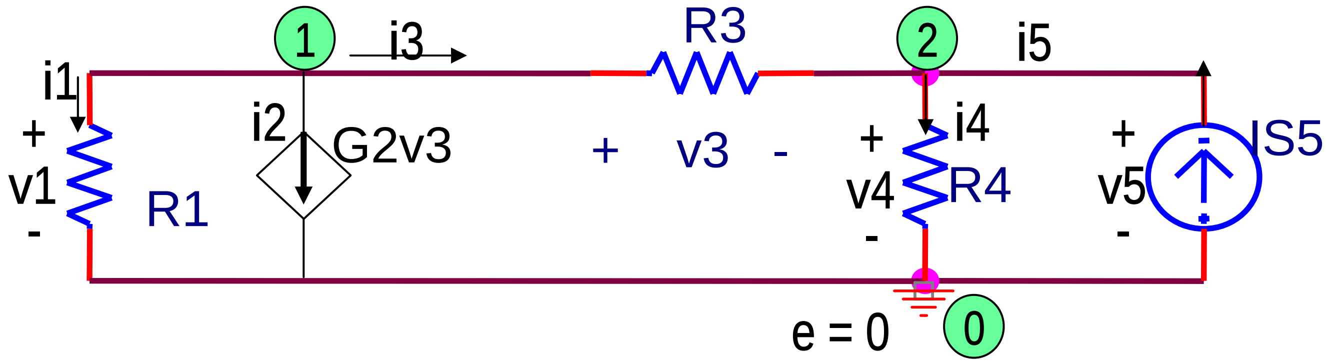
Modified Nodal Analysis (MNA)

- **A more <u>compact</u> formulation**
- **In MNA, every element is in conductance form!**

- **We'll review the steps how MNA is done.**

- **Introduced by McCalla, Nagel, Rohrer, Ruehli, Ho (1975)**

Nodal Analysis



Step 1: **Write KCL:** **i1 + i2 + i3 = 0** **(node 1)**

-i3 + i4 – i5 = 0 **(node 2)**

Step 2: **Substitute branch equations to rewrite KCL in branch voltages:**

$$\frac{1}{R1}v1 + G2*v3 + \frac{1}{R3}v3 = 0 \qquad (1)$$

$$-\frac{1}{R3}v3 + \frac{1}{R4}v4 = IS5 \qquad (2)$$

Nodal Analysis

Step 3: Substitute branch voltages by nodal voltages (using KVL):

$$\frac{1}{R1}e1 + G2(e1 - e2) + \frac{1}{R3}(e1 - e2) = 0 \qquad (1)$$

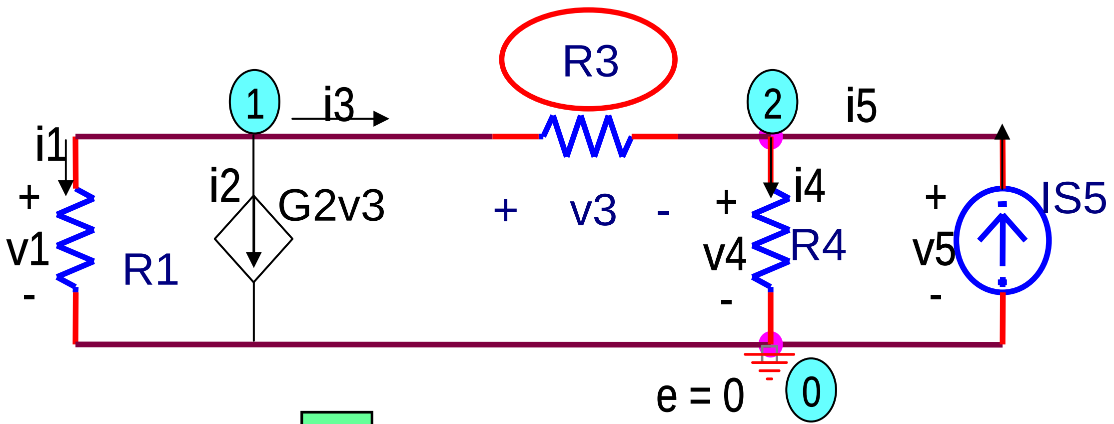
$$-\frac{1}{R3}(e1-e2) + \frac{1}{R4}e2 = IS5 \qquad (2)$$

Put in matrix form

$$\begin{bmatrix} \frac{1}{R_1} + G_2 + \frac{1}{R_3} & -G_2 - \frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_4} + \frac{1}{R_3} \end{bmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 0 \\ I_{S5} \end{pmatrix}$$

$$Y_n e = S$$

Regularity in MNA Matrix



Stamping

$$
\begin{array}{c c}
 & \begin{array}{cc} \quad (1) \quad & \quad (2) \quad \end{array} \\
\begin{array}{c} (1) \\ \\ (2) \end{array} &
\begin{bmatrix}
\frac{1}{R_1} + G_2 + \frac{1}{R_3} & -G_2 - \frac{1}{R_3} \\
-\frac{1}{R_3} & \frac{1}{R_4} + \frac{1}{R_3}
\end{bmatrix}
\end{array}
$$

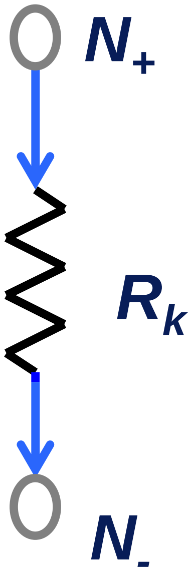
Coefficient matrix

- **Each element contributes (in conductance form) only to the entries with row-column positions corresponding to the node numbers.**
- **Such a regular format is called a "stamp"**

Resistor Stamp

SPICE Netlist Format (R)

R_k N_+ N- value_of_R_k



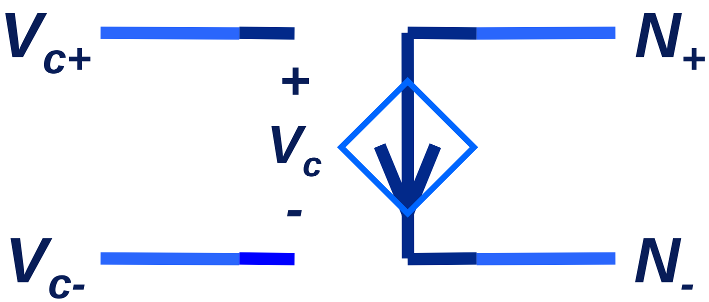
$$
\begin{array}{c|ccccc}
 & & N+ & & N- & \\
 & & \vdots & & \vdots & \\
N+ & \cdots & \frac{1}{R_k} & \cdots & -\frac{1}{R_k} & \cdots \\
 & & \vdots & & \vdots & \\
N- & \cdots & -\frac{1}{R_k} & \cdots & \frac{1}{R_k} & \cdots \\
 & & \vdots & & \vdots &
\end{array}
$$

VCCS Stamp

SPICE Netlist Format (VCCS)

G_k N_+ $N-$ NC_+ $NC-$ $value\_of\_G_k$

Similar to a resistor; but note that the row/col indices are different.



$$
\begin{array}{c|ccccc}
 & & Nc+ & & Nc- & \\
 & & \vdots & & \vdots & \\
N+ & \cdots & G_k & \cdots & -G_k & \cdots \\
 & & \vdots & & \vdots & \\
N- & \cdots & -G_k & \cdots & G_k & \cdots \\
 & & \vdots & & \vdots &
\end{array}
$$

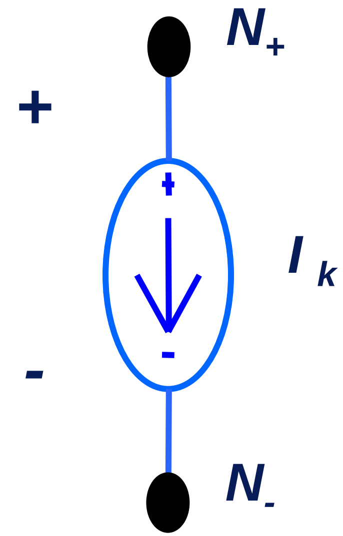
Current Source Stamp

SPICE Netlist Format (Current Source)

ISK   N+   N-   value_of_I_k

Note the signs in this case!

$$\begin{array}{cc} \begin{pmatrix} \vdots \\ -I_k \\ \vdots \\ +I_k \\ \vdots \end{pmatrix} & \begin{array}{c} \\ N+ \\ \\ N- \\ \\ \end{array} \end{array}$$



Right-Hand Side (RHS)

Relation between STA and NA

$$\begin{array}{cc} \begin{matrix} K_i^{-1} \\ \\ \end{matrix} & \begin{pmatrix} K_i & -K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{pmatrix} \begin{pmatrix} i \\ v \\ e \end{pmatrix} = \begin{pmatrix} S \\ 0 \\ 0 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} \begin{matrix} -A \\ \\ \end{matrix} & \begin{pmatrix} I & -K_i^{-1}K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{pmatrix} \begin{pmatrix} i \\ v \\ e \end{pmatrix} = \begin{pmatrix} K_i^{-1}S \\ 0 \\ 0 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} I & -K_i^{-1}K_v & 0 \\ 0 & I & -A^T \\ 0 & AK_i^{-1}K_v & 0 \end{pmatrix} \begin{pmatrix} i \\ v \\ e \end{pmatrix} = \begin{pmatrix} K_i^{-1}S \\ 0 \\ -AK_i^{-1}S \end{pmatrix}$$

Relation between STA and NA

$$-AK_i^{-1}K_v \quad \begin{pmatrix} I & -K_i^{-1}K_v & 0 \\ 0 & I & -A^T \\ 0 & AK_i^{-1}K_v & 0 \end{pmatrix} \begin{pmatrix} i \\ v \\ e \end{pmatrix} = \begin{pmatrix} K_i^{-1}S \\ 0 \\ -AK_i^{-1}S \end{pmatrix} \Rightarrow$$

Tableau Matrix

$$\Rightarrow \begin{pmatrix} I & -K_i^{-1}K_v & 0 \\ 0 & I & -A^T \\ 0 & 0 & AK_i^{-1}K_vA^T \end{pmatrix} \begin{pmatrix} i \\ v \\ e \end{pmatrix} = \begin{pmatrix} K_i^{-1}S \\ 0 \\ -AK_i^{-1}S \end{pmatrix}$$

Yn ***Is***

MNA

$$\Rightarrow \boxed{Y_n e = Is}$$

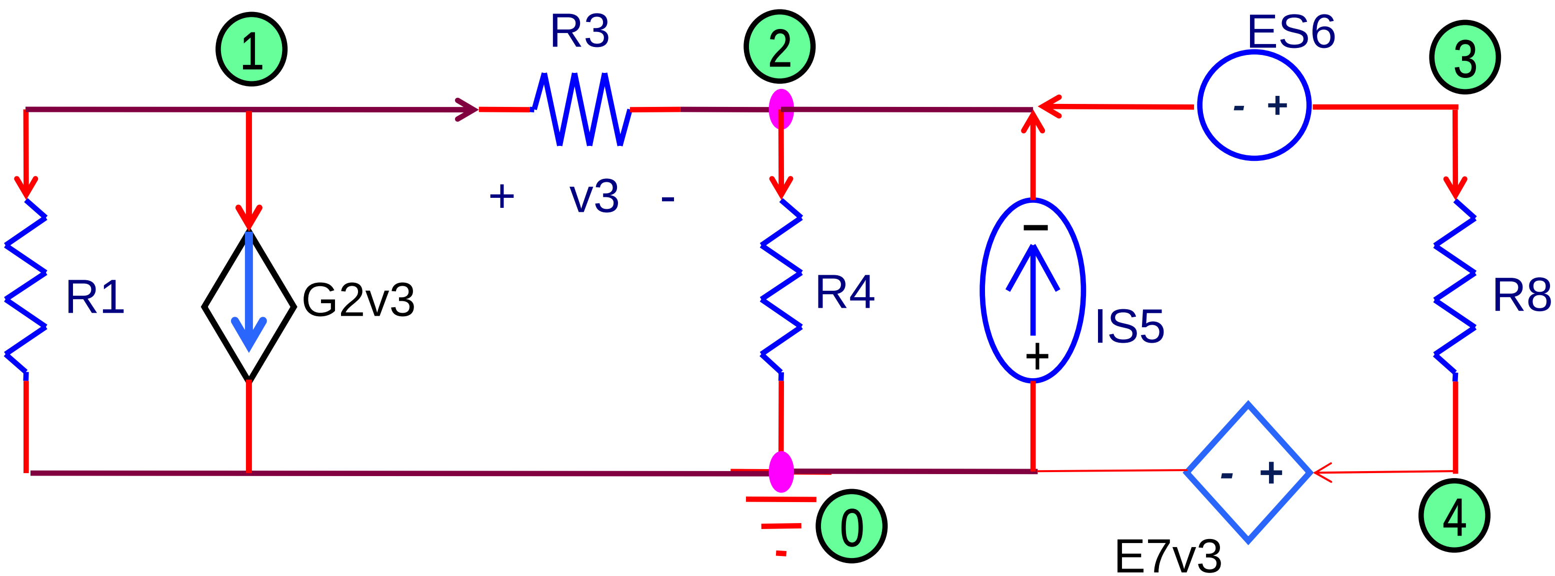
After solving *e*, we get *v*, then get ***i***.

Nodal Analysis -- Advantages & Problem

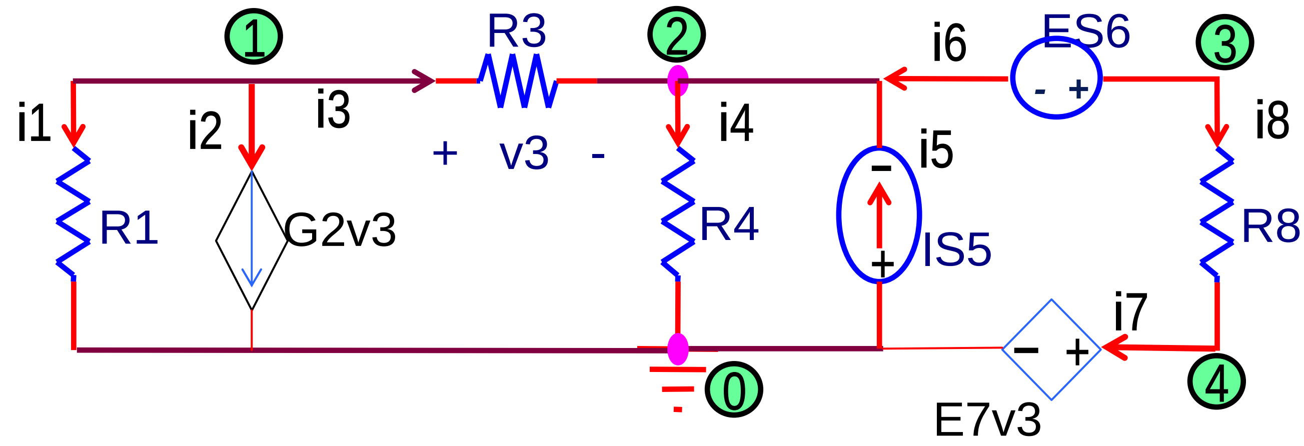
- **Advantages**:
  - **Circuit equations can be assembled by stamping**
  - ***Yn* is sparse (but not as sparse as STA) and small (*nxn*), smaller than STA (*n + 2b*n + 2b*)**
  - ***Yn* has non-zero diagonal entries and is often diagonally dominant**
- **Problem**:

  Nodal Analysis cannot handle the following
  - **Floating independent voltage source (not connected to ground)**
  - **VCVS (E-ELEMENT)**
  - **CCCS (F-ELEMENT)**
  - **(VCCS ok!) (G-ELEMENT)**
  - **CCVS (H-ELEMENT)**

Modified Nodal Analysis (MNA)



Modified Nodal Analysis (MNA)



<u>Step 1</u>: Write KCL

$i1 + i2 + i3 = 0$ **(1)**

$-i3 + i4 - i5 - i6 = 0$ **(2)**

$i6 + i8 = 0$ **(3)**

$i7 - i8 = 0$ **(4)**

Modified Nodal Analysis (MNA)

<u>Step 2</u>: Substitute branch currents by branch voltages

$$\frac{1}{R1}v1+G2v3+\frac{1}{R3}v3=0 \qquad (1)$$

$$-\frac{1}{R3}v3+\frac{1}{R4}v4-i6=IS5 \qquad (2)$$

$$i6+\frac{1}{R8}v8=0 \qquad (3)$$

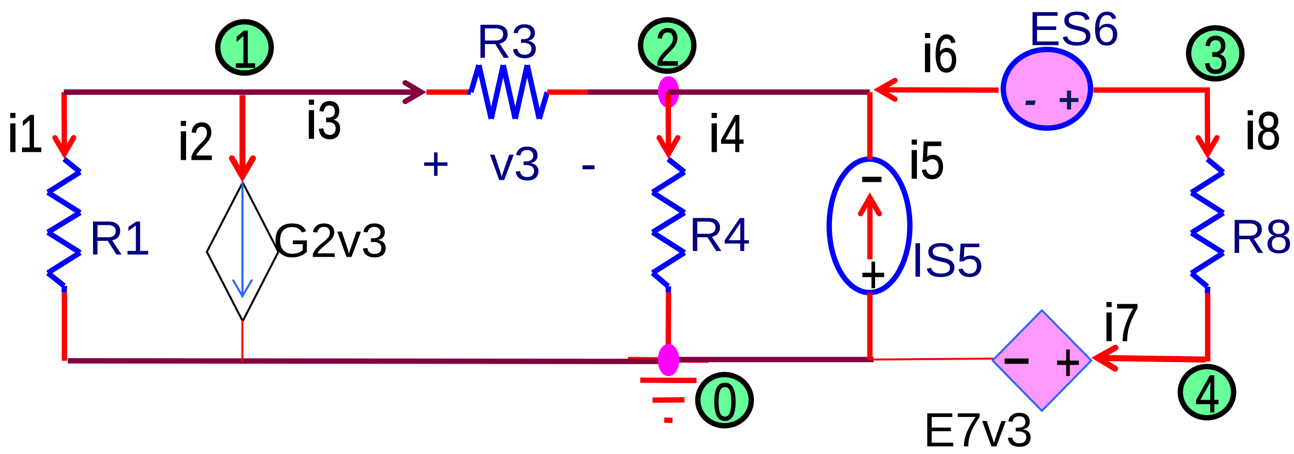
$$i7-\frac{1}{R8}v8=0 \qquad (4)$$

Modified Nodal Analysis (MNA)

<u>Step 3</u>: Write down unused branch equations

$$v_6 = ES_6 \qquad (4)$$

$$v_7 - E_7 v_3 = 0 \qquad (5)$$



Modified Nodal Analysis (MNA)

Step 4: Substitute branch voltages by nodal voltages

$$\frac{1}{R1}e1+G2(e1-e2)+\frac{1}{R3}(e1-e2)=0 \qquad (1)$$

$$-\frac{1}{R3}(e1-e2)+\frac{1}{R4}e2-i6=IS5 \qquad (2)$$

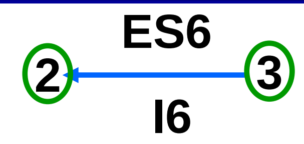
$$i6+\frac{1}{R8}(e3-e4)=0 \qquad (3)$$

$$i7-\frac{1}{R8}(e3-e4)=0 \qquad (4)$$

$$(e3-e2)=ES6 \qquad (5)$$

$$e4-E7(e1-e2)=0 \qquad (6)$$

Modified Nodal Analysis (MNA)



$$
\begin{array}{c}
\text{node-1} \\ \text{node-2} \\ \text{node-3} \\ \text{node-4} \\ \text{branch-6} \\ \text{branch-7}
\end{array}
\begin{array}{c}
\begin{array}{cccccc} \text{node-1} & \text{node-2} & \text{node-3} & \text{node-4} & \text{branch} & \text{branch} \end{array} \\
\left[\begin{array}{cccc:cc}
\frac{1}{R1}+G2+\frac{1}{R3} & -G2-\frac{1}{R3} & 0 & 0 & 0 & 0 \\
-\frac{1}{R3} & \frac{1}{R3}+\frac{1}{R4} & 0 & 0 & -1 & 0 \\
0 & 0 & \frac{1}{R8} & -\frac{1}{R8} & 1 & 0 \\
0 & 0 & -\frac{1}{R8} & \frac{1}{R8} & 0 & 1 \\
\hdashline
0 & -1 & 1 & 0 & 0 & 0 \\
E7 & -E7 & 0 & -1 & 0 & 0
\end{array}\right]
\end{array}
\begin{bmatrix} e1 \\ e2 \\ e3 \\ e4 \\ i6 \\ i7 \end{bmatrix}
=
\begin{bmatrix} 0 \\ IS5 \\ 0 \\ 0 \\ ES6 \\ 0 \end{bmatrix}
\begin{array}{l} \\ \\ \\ \\ \text{branch-6} \\ \text{branch-7} \end{array}
$$

$$
\left(\begin{array}{c:c} Y_n & B \\ \hdashline C & 0 \end{array}\right)
\begin{pmatrix} e \\ i \end{pmatrix} = RHS
$$

node voltages

some branch currents

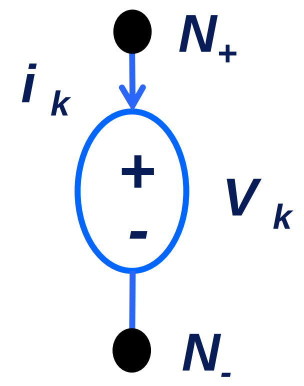
Voltage Source Stamp

SPICE Netlist Format (Floating voltage source)

V_K N+ N- value_of_Vk

current introduced!

	N+	N-	i_k	RHS
N+	0	0	1	0
N-	0	0	-1	0
branch k	1	-1	0	V_k



CCCS Stamp

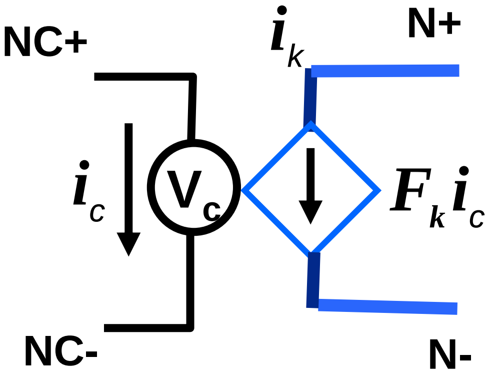
SPICE Netlist Format (CCCS)

F_K N+ N- Vname value_of_F_K

Vname NC+ NC- value

	N_+	N-	NC_+	NC -	i_c
N_+					F_k
N-					$-F_k$
NC+					1
NC-					-1
br Vc			1	-1	

$$\text{RHS} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_c \end{bmatrix}$$



* If ‘Vname’ is used as a CC for multiple times, it is stamped **only once** though!

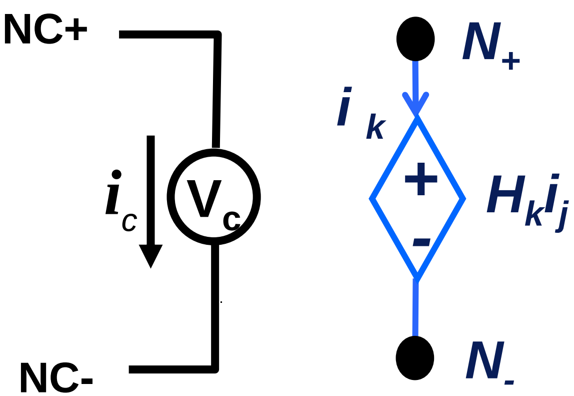
CCVS Stamp

SPICE Netlist Format (CCVS)

H_K N+ N- Vname value_of_H_K

Vname NC+ NC- value

	N_+	N-	NC_+	NC-	i_k	i_c	RHS
N_+					1		0
N-					-1		0
NC+						1	0
NC-						-1	0
br-k	1	-1				$-H_K$	0
br-c			1	-1			V_c



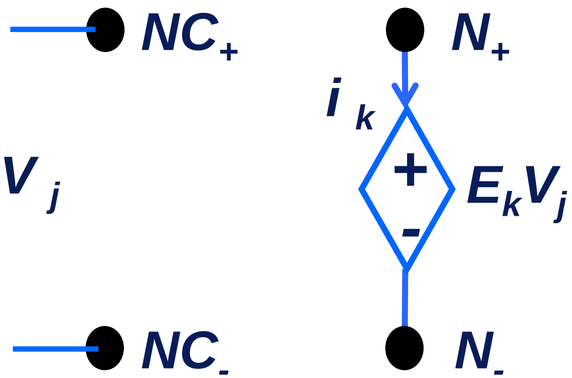
* If 'Vname' is used as a CC for multiple times, it is stamped **only once** though!

VCVS Stamp

SPICE Netlist Format (VCVS)

E_K N+ N- NC+ NC- value_of_E_K

	N_+	N-	NC_+	NC-	i_k
N_+					1
N-					-1
NC_+					
NC-					
br k	1	-1	$-E_k$	E_k	



General Rules for MNA

- A branch current is introduced as an additional variable for a voltage source or an inductor
- For current sources, resistors, conductance and capacitors, the branch current is introduced only if
  - Any circuit element depends on that branch current; or
  - The branch current is requested as an output.

Modified Nodal Analysis (MNA)

Advantages of MNA

- **MNA can be applied to any circuit**
- **MNA equations can be assembled “directly” from a circuit description (e.g. netlist)**

Problem

- **Sometimes zeros appear on the main diagonal; causing some principle minors to be singular (numerical instability.)**

Summary

- **KVL/KCL + Circuit Element Equations**
- **Equations formulation: STA and MNA**
- **MNA was implemented in most simulators (SPICE)**
- **Element stamps**

- **A key observation:**
  - **Circuit matrix structure will not change! (exploited by SPICE for speedup – symbolic factorization)**

Assignment 3

- **Implement a netlist parser that reads a simple netlist with the following elements**
  - **R**
  - **Vsource, Isource**
  - **VCVS, CCCS, VCCS, CCVS**

  Print the stamps and the RHS with row and column indices.

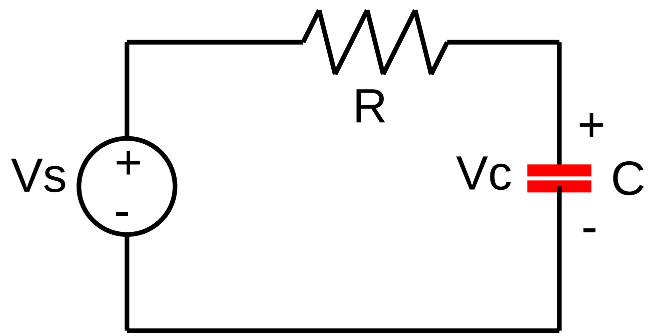
Part 2.
Dynamic Element Stamping

Outline

- **Discretization Formulas for d/dt**
- **Element Stamps for Linear Capacitors and Inductors**

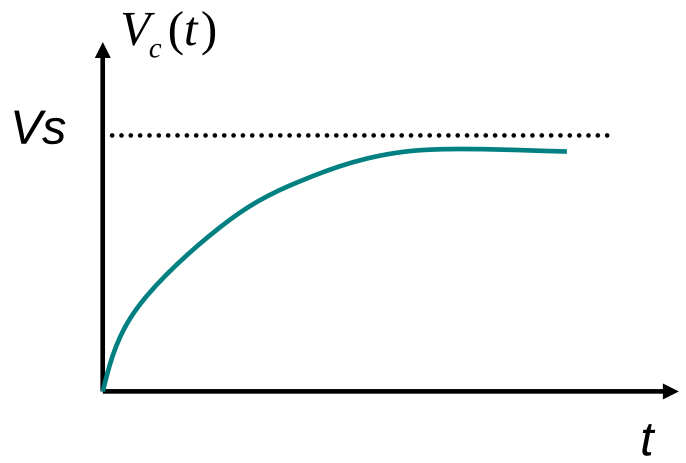
Circuit with dynamic element

$$RC\frac{dV_c}{dt} + V_c = V_s, \quad V_c(0) = 0$$



Analytical solution

$$V_c(t) = V_s\left(1 - e^{-\frac{t}{\tau}}\right), \qquad \tau = RC$$



- How to solve it numerically?

Numerical Solution

$$\boxed{\frac{\mathrm{dV}}{\mathrm{dt}}} + \mathrm{V} = \mathrm{V_s}, \quad V(0) = 0$$

Assuming τ = RC = 1

- Replace the derivative by difference

$$\boxed{\frac{V(t+h) - V(t)}{h}} + V(t) = V_s$$

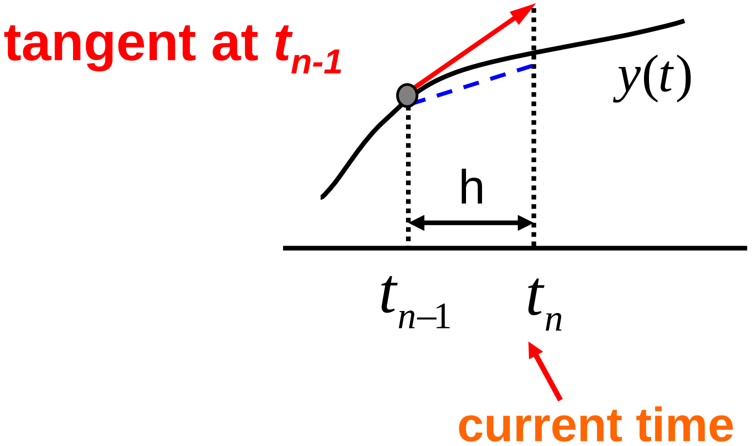
h = time step (small)

There are many ways to do discretization.

$$\boxed{V(t+h) = V(t) + h[V_s - V(t)]}$$

Becomes iteration

Forward Euler (FE)



$$\dot{y}(t) = \frac{dy(t)}{dt} = f(y(t))$$

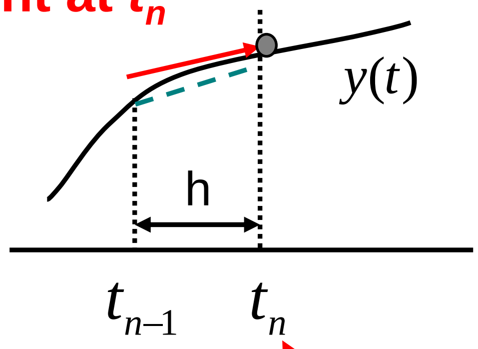
$$\frac{y(t_n) - y(t_{n-1})}{h} \approx \dot{y}(t_{n-1}) = f(y(t_{n-1}))$$

$$\boxed{y(t_n) = y(t_{n-1}) + h \bullet f\left(y(t_{n-1})\right)}$$

Direct iteration; (no linear / nonlinear solve involved)

Backward Euler (BE)

$$\boxed{\dot{y}(t) = \frac{dy(t)}{dt} = f(y(t))}$$



$$\frac{y(t_n) - y(t_{n-1})}{h} \approx \dot{y}(t_n) = f(y(t_n))$$

$$\boxed{y(t_n) = y(t_{n-1}) + h \bullet f\left(y(t_n)\right)}$$

$y(t_n)$ appears on both sides of the equation; need to solve *$y(t_n)$* by iterations if f(·) is nonlinear.

FE and BE

Forward Euler: $$\boxed{y(t_n) = y(t_{n-1}) + h \bullet f\left(y(t_{n-1})\right)}$$

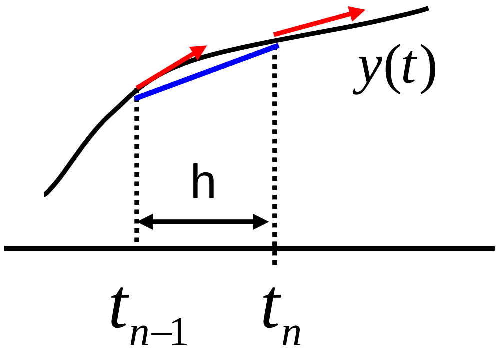
Backward Euler: $$\boxed{y(t_n) = y(t_{n-1}) + h \bullet f\left(y(t_n)\right)}$$

- **Although B.E. requires more computation, it is preferred in practice because it is more numerically stable.**
- **Will discuss on this later.**

Trapezoidal Rule (TR)

$$\dot{y}(t) = \frac{dy(t)}{dt} = f(y(t))$$

tangents at both t_{n-1} and t_n



$$\boxed{\frac{y(t_n) - y(t_{n-1})}{h} \approx \frac{1}{2}\left[\dot{y}(t_n) + \dot{y}(t_{n-1})\right]}$$

slope of secant

averaged tangent

$$\frac{y(t_n) - y(t_{n-1})}{h} \approx \frac{1}{2}\left[f(y(t_n)) + f(y(t_{n-1}))\right]$$

$$\boxed{y(t_n) = y(t_{n-1}) + \frac{h}{2}\left[f(y(t_n)) + f(y(t_{n-1}))\right]}$$

Again, requires solving $y(t_n)$ from a set of nonlinear equations

FE, BE and TR

$$\dot{y}(t) = \frac{dy(t)}{dt} = f(y(t))$$

Forward Euler: $$y(t_n) = y(t_{n-1}) + h \bullet f\left(y(t_{n-1})\right)$$

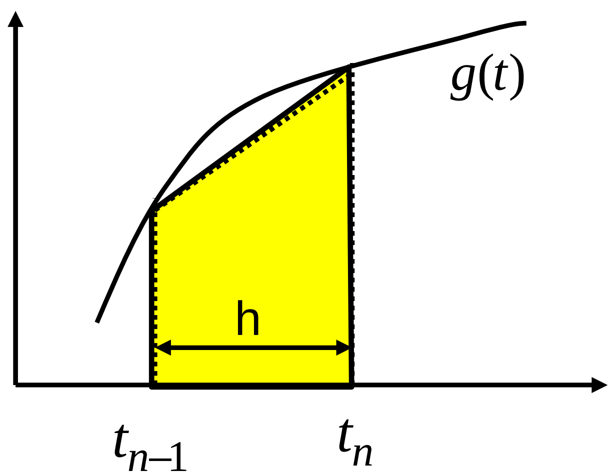
Backward Euler: $$y(t_n) = y(t_{n-1}) + h \bullet f\left(y(t_n)\right)$$

Trapezoidal Rule: $$y(t_n) = y(t_{n-1}) + \frac{h}{2}\left[f(y(t_n)) + f(y(t_{n-1}))\right]$$

- **<u>Trapezoidal rule</u> has even better numerical property than B.E.**

Interpretation of Trapezoidal Rule

- Trapezoidal rule comes from numerical integration – computing the area under a curve



$$\dot{x}(t) = g(t)$$

area under the curve

$$x(t_n) = x(t_{n-1}) + \int_{t_{n-1}}^{t_n} g(\tau)d\tau$$

$$x(t_n) \approx x(t_{n-1}) + \frac{h}{2}\left[g(t_n) + g(t_{n-1})\right]$$

approximated by a trapezoidal

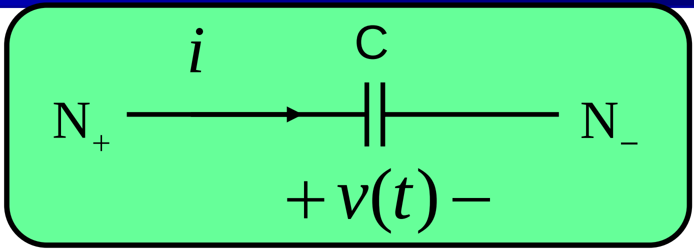
$$\boxed{\frac{x(t_n) - x(t_{n-1})}{h} \approx \frac{1}{2}\left[\dot{x}(t_n) + \dot{x}(t_{n-1})\right]}$$

Trapezoidal Rule

Stamps of Dynamic Elements

- **<u>Stamps for dynamic elements</u> can be derived from a discretization method:**
  - **Capacitor (C)**
  - **Inductor (L)**
  - **Others ...**

Capacitor Stamp



$$i(t) = C\frac{dv(t)}{dt}$$

discretized by B.E.

$$i(t) = \frac{C}{h}\left[v(t) - v(t-h)\right] = \frac{C}{h}v(t) - \boxed{\frac{C}{h}v(t-h)}$$

treated as source

NA Stamp

	N_+	N_-	RHS
N_+	$\frac{C}{h}$	$-\frac{C}{h}$	$\frac{C}{h}v(t-h)$
N_-	$-\frac{C}{h}$	$\frac{C}{h}$	$-\frac{C}{h}v(t-h)$

MNA Stamp

	N_+	N_-	i	RHS
N_+			1	0
N_-			-1	0
branch C	$\frac{C}{h}$	$-\frac{C}{h}$	-1	$\frac{C}{h}v(t-h)$

$$\boxed{\frac{C}{h}v(t) - i(t) = \frac{C}{h}v(t-h)}$$

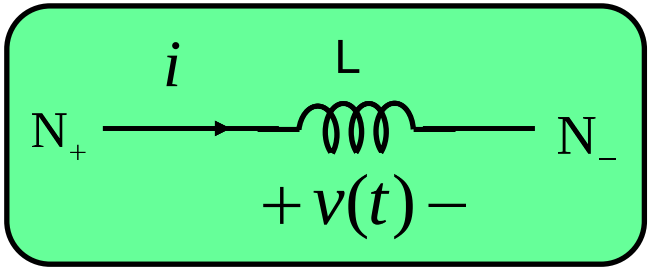
Inductor Stamp

discretized by B.E.

$$v(t) = L\frac{di}{dt} \approx L\frac{i(t)-i(t-h)}{h}$$

$$\boxed{v(t) - \frac{L}{h} i(t) = -\frac{L}{h} i(t-h)}$$

(must introduce current)



$$\begin{array}{r|ccc:c}
 & N_+ & N_- & i & \text{RHS} \\
\hline
N_+ & & & 1 & 0 \\
N_- & & & -1 & 0 \\
\textbf{branch L} & 1 & -1 & -\frac{L}{h} & -\frac{L}{h} i(t-h)
\end{array}$$

MNA Stamp

Stamps for C & L

NA Stamp for C

$$
\begin{array}{c|cc:c}
 & N_+ & N_- & \text{RHS} \\
\hline
N_+ & \frac{C}{h} & -\frac{C}{h} & \frac{C}{h}v(t-h) \\
N_- & -\frac{C}{h} & \frac{C}{h} & -\frac{C}{h}v(t-h)
\end{array}
$$

MNA Stamp for C

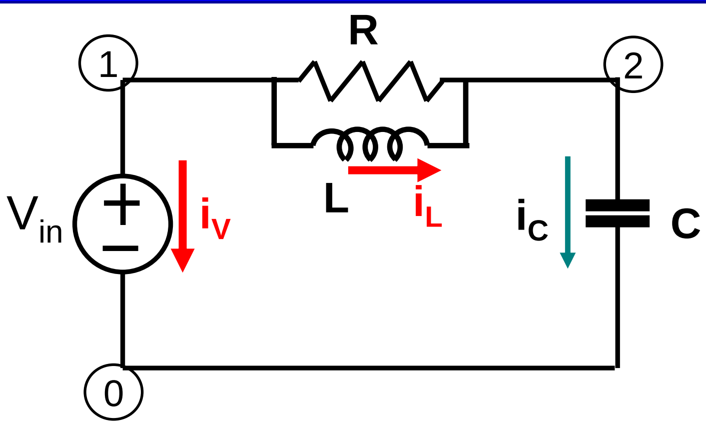
$$
\begin{array}{c|ccc:c}
 & N_+ & N_- & i & \text{RHS} \\
\hline
N_+ & & & 1 & 0 \\
N_- & & & -1 & 0 \\
 & \frac{C}{h} & -\frac{C}{h} & -1 & \frac{C}{h}v(t-h)
\end{array}
$$

MNA Stamp for L

$$
\begin{array}{c|ccc:c}
 & N_+ & N_- & i & \text{RHS} \\
\hline
N_+ & & & 1 & 0 \\
N_- & & & -1 & 0 \\
 & 1 & -1 & -\frac{L}{h} & -\frac{L}{h}i(t-h)
\end{array}
$$

Note that: Stamps for C or L depend on the discretization method used!

A Circuit Example



$$i_c(t) = \frac{C}{h}\left[v_C(t) - v_C(t-1)\right]$$

$$= \frac{C}{h}\left[v_2(t) - v_0(t)\right] - = \frac{C}{h}\left[v_2(t-1) - v_0(t-1)\right]$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$v_1(t) - v_2(t) = \frac{L}{h}\left[i_L(t) - i_L(t-1)\right]$$

		0	1	2	3 (i_L)	4 (i_V)	RHS
	0	$\frac{C}{h}$	0	$-\frac{C}{h}$	0	-1	$-\frac{C}{h}v_c(t-h)$
	1	0	$\frac{1}{R}$	$-\frac{1}{R}$	1	1	0
	2	$-\frac{C}{h}$	$-\frac{1}{R}$	$\left(\frac{1}{R}+\frac{C}{h}\right)$	-1		$\frac{C}{h}v_c(t-h)$
branch L	3	0	1	-1	$-\frac{L}{h}$		$-\frac{L}{h}i_L(t-h)$
branch V_{in}	4	-1	1				V_{in}

Assignment 4

- **Derive the stamps for C and L using the Trapezoidal Rule (both in MNA).**

Classical Papers

1. **G.D. Hachtel, R.K. Brayton and F.G. Gustavson, "The sparse tableau approach to network analysis and design," *IEEE Trans. Circuit Theory*, vol.CT-18, Jan. 1971, pp. 101-119.**
2. **C.W. Ho, A.E. Ruehli and P.A. Brennan, "The modified nodal approach to network analysis", *IEEE Trans. Circuits and Systems*, CAS-22, June 1975, pp. 504-509**