# Chapter 3 <br> Arithmetic for Computers 

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## Arithmetic for Computers

- Operations on integers
- Addition and subtraction
- Multiplication and division
- Dealing with overflow
- Floating-point real numbers
- Representation and operations
- ALU: Arithmetic Logic Unit
- HW performs addition, subtraction, and usually logical operations such as AND and OR


## Integer Addition

- Example: 7 + 6


Overflow if result out of range (2's complement)

- Adding +ve and -ve operands, no overflow
- Adding two +ve operands
- Overflow if result sign is 1
- Adding two -ve operands
- Overflow if result sign is 0


## Integer Subtraction

- Add negation of second operand
- Example: 7 - $6=7+(-6)$
+7: $00000000 \ldots 00000111$

| $-6:$ | $11111111 \ldots 11111010$ |
| :--- | :--- |
| $+1:$ | $00000000 \ldots 00000001$ |

- Overflow if result out of range (2's complement)
- Subtracting two +ve or two -ve operands, no overflow
- Subtracting +ve from -ve operand
: Overflow if result sign is 0
- Subtracting -ve from +ve operand
- Overflow if result sign is 1


## Dealing with Overflow

- Some languages ignore overflow
- E.g., C
- Use MIPS addu, addiu, subu instructions
- Checked by programmer
- Other languages require raising an exception
- E.g., Ada, Fortran
- Use MIPS add, addi, sub instructions
- On overflow, invoke exception handler
- Save PC in exception program counter (EPC) register
- Jump to predefined handler address
- mfc0 (move from coprocessor reg) instruction can retrieve EPC value, to return after corrective action


## Dealing with Overflow

- add rd,rs,rt
if (NotWordValue(GPR[rs]) or NotWordValue(GPR[rt])) then UndefinedResult() endif temp $\leftarrow$ GPR[rs] + GPR[rt]
if (32_bit_arithmetic_overflow) then
SignalException(IntegerOverflow)
else
GPR[rd] $\leftarrow$ sign_extend(temp $31 . .0)$
endif
- addu rd,rs,rt
if (NotWordValue(GPR[rs]) or NotWordValue(GPR[rt])) then UndefinedResult() endif temp $\leftarrow$ GPR[rs] + GPR[rt]
GPR $\left[\right.$ rd] $\leftarrow$ sign_extend $\left(\right.$ temp $\left.{ }_{31 \ldots .0}\right)$


## Arithmetic for Multimedia

- Graphics and media processing operates on vectors of 8 -bit and 16-bit data
- 8 bits: byte, 16 bits: halfword
- Use 64-bit adder, with partitioned carry chain

Operate on $8 \times 8$-bit, $4 \times 16$-bit, or $2 \times 32$-bit vectors

- SIMD (single-instruction, multiple-data), vector or Sub -Word Parallel
- Saturating operations
- On overflow, result is set to largest representable value (largest positive or most negative number)
C.f. $2 s$-complement modulo arithmetic
- E.g., clipping in audio, saturation in video
- Multimedia extensions to ISAs
- MDMX (MIPS Digital Media eXtension), c.f. Intel SSE


## Multiplication

## - Start with long-multiplication approach



## Length of product is the sum of operand lengths



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## Multiplication Hardware



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## Optimized Multiplier

- Perform steps in parallel: add/shift

- One cycle per partial-product addition
- That's ok, if frequency of multiplications is low


## Faster Multiplier

- Uses multiple adders
- Parallel tree, $\log _{2}(32)$ or 5 32-bit add time
- Cost/performance tradeoff


Product63 Product62


Product1 Product0

- Can be pipelined
- Several multiplication performed in parallel
- More resource


## MIPS Integer Multiplication

- Two special 32-bit registers for 64-bit product
- HI: most-significant 32 bits
- LO: least-significant 32-bits
- Instructions
- mult rs, rt / multu rs, rt (2 operands)
- 64-bit product in HI/LO
- mfhi rd / mflo rd (1 operand)
- Move from HI/LO to rd
- Can test HI value to see if product overflows 32 bits
- mul rd, rs, rt (32b result)

Least-significant 32 bits of product -> rd

- Pseudoinstruction


## Division



- Check for 0 divisor
- Long division approach
- If divisor $\leq$ dividend bits
- 1 bit in quotient, subtract
- Otherwise
- 0 bit in quotient, bring down next dividend bit
- Restoring division
- Do the subtract, and if remainder goes < 0, add divisor back
- Signed division
- Divide using absolute values
- Adjust sign of quotient and remainder as required


## Division Hardware



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## Optimized Divider



- One cycle per partial-remainder subtraction
- Looks a lot like a multiplier!
- Same hardware can be used for both


## Faster Division

- Can't use parallel hardware as in multiplier
- Subtraction is conditional on sign of remainder
- As a rule, the dividend and remainder must have the same sign, no matter what the signs of the divisor and quotient
- Faster dividers (e.g. SRT division) generate multiple quotient bits per step
- A typical value today is 4 bits
- The key is guessing the value to subtract
- Still require multiple steps


## MIPS Division

- Use HI/LO registers for result as multiplication
- HI: 32-bit remainder
- LO: 32-bit quotient
- Instructions
- div rs, rt / divu rs, rt
- No overflow or divide-by-0 checking

Software must perform checks if required

- Use mfhi, mflo to access result
- div rd, rs, rt: pseudoinstruction


## Floating Point

- Representation for non-integral numbers
- Including very small and very large numbers
- Like scientific notation
$-2.34 \times 10^{56}$ normalized
- +0.002 × 10-4
- $+987.02 \times 10^{9}$
- In binary
- $\pm 1 . x x x x x x x_{2} \times 2 y y y$
- Types float and double in C


## Floating Point Standard

- Defined by IEEE Std. 754-1985
- Developed in response to divergence of representations
- Portability issues for scientific code
- Now almost universally adopted
- Two representations
- Single precision (32-bit)
- Double precision (64-bit)


## IEEE Floating-Point Format

single: 8 bits double: 11 bits
single: 23 bits
double: 52 bits

## S Exponent Fraction

$$
x=(-1)^{s} \times(1+\text { Fraction }) \times 2^{\text {(Exponent-Bias) })}
$$

- S : sign bit ( $0 \Rightarrow$ non-negative, $1 \Rightarrow$ negative )
- Normalize significand: $1.0 \leq \mid$ significand $\mid<2.0$
- Always has a leading pre-binary-point 1 bit, so no need to represent it explicitly (hidden bit)
- Significand is Fraction with the "1." To restore: 1+Fraction
- Exponent: excess representation: actual exponent + Bias
- Ensures exponent is unsigned, biased notation
- IEEE 754's bias

Single: Bias $=127=2^{7}-1 ; \quad$ Double: Bias $=1203=2^{10}-1$

- Bias is $2^{(n-1)}-1$, c.f. conventional $2^{(n-1)}$


## Floating-Point Example

- Represent -0.75
$-0.75=(-1)^{1} \times 1.1_{2} \times 2^{-1}$
- $S=1$
- Fraction = 1000...00
- Exponent = -1 + Bias
- Single: $-1+127=126=0111110_{2}$
- Double: $-1+1023=1022=0111111111_{2}$
- Single: 1011111101000... 00

■ Double: 1011111111101000... 00

## Floating-Point Example

- What number is represented by the single -precision float
11000000101000... 00
- S = 1
- Fraction = 01000...00
- Exponent $=10000001_{2}=129$
- $\mathrm{x}=(-1)^{1} \times\left(1+01_{2}\right) \times 2^{(129-127)}$
$=(-1) \times 1.25 \times 2^{2}$
$=-5.0$


## Single-Precision Range

- Exponents 00000000 and 11111111 reserved
- Smallest value
- Exponent: 00000001
$\Rightarrow$ actual exponent $=1-127=-126$
- Fraction: $000 . . .00 \Rightarrow$ significand $=1.0$
- $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- Largest value
- exponent: 11111110
$\Rightarrow$ actual exponent $=254-127=+127$
- Fraction: $111 \ldots 11 \Rightarrow$ significand $\approx 2.0$
$\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$


## Double-Precision Range

. Exponents 0000... 00 and 1111... 11 reserved

- Smallest value
- Exponent: 00000000001
$\Rightarrow$ actual exponent $=1-1023=-1022$
- Fraction: 000... $00 \Rightarrow$ significand $=1.0$
$- \pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- Largest value
- Exponent: 11111111110
$\Rightarrow$ actual exponent $=2046-1023=+1023$
- Fraction: $111 \ldots 11 \Rightarrow$ significand $\approx 2.0$
$- \pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$


## Floating-Point Precision

- Relative precision
- All fraction bits are significant
- Single: approx 2-23
- Equivalent to $23 \times \log _{10} 2 \approx 23 \times 0.3 \approx 6$ decimal digits of precision
- Double: approx $2^{-52}$

Equivalent to $52 \times \log _{10} 2 \approx 52 \times 0.3 \approx 16$ decimal digits of precision

## Denormal Numbers

- Exponent $=000 \ldots 0 \Rightarrow$ hidden bit is 0

$$
x=(-1)^{S} \times(0+\text { Fraction }) \times 2^{- \text {Bias }}
$$

Smaller than normal numbers

- allow for gradual underflow, with diminishing precision

Denormal with fraction $=000 \ldots 0$

$$
\begin{gathered}
x=(-1)^{S} \times(0+0) \times 2^{- \text {Bias }}= \pm 0.0 \\
\begin{array}{c}
\text { Two representations } \\
\text { of } 0.0!
\end{array}
\end{gathered}
$$

## Infinities and NaNs

- Exponent = 111...1, Fraction = 000... 0
- Infinity
- Can be used in subsequent calculations, avoiding need for overflow check
- Exponent = 111...1, Fraction $\neq 000 . . .0$
- Not-a-Number (NaN)
- Indicates illegal or undefined result e.g., 0.0 / 0.0
- Can be used in subsequent calculations


## Floating-Point Addition

- Consider a 4-digit decimal example
- $9.999 \times 10^{1}+1.610 \times 10^{-1}$

1. Align decimal points

Align the number with smaller exponent, match the larger exponent $9.999 \times 10^{1}+0.016 \times 10^{1}$
2. Add significands
$9.999 \times 10^{1}+0.016 \times 10^{1}=10.015 \times 10^{1}$
3. Normalize result \& check for over/underflow
$1.0015 \times 10^{2}$
4. Round and renormalize if necessary
$1.002 \times 10^{2}$

## Floating-Point Addition

- Now consider a 4-digit binary example
$=1.000_{2} \times 2^{-1}+-1.110_{2} \times 2^{-2}(0.5+-0.4375)$

1. Align binary points

Shift number with smaller exponent, match the larger exponent $1.000_{2} \times 2^{-1}+-0.111_{2} \times 2^{-1}$
2. Add significands
$1.000_{2} \times 2^{-1}+-0.111_{2} \times 2^{-1}=0.001_{2} \times 2^{-1}$
3. Normalize result \& check for over/underflow
$1.000_{2} \times 2^{-4}$, with no over/underflow
4. Round and renormalize if necessary
$1.000_{2} \times 2^{-4}$ (no change) $=0.0625$

## FP Adder Hardware

- Much more complex than integer adder
- Doing it in one clock cycle would take too long
- Much longer than integer operations
- Slower clock would penalize all instructions
- FP adder usually takes several cycles
- Can be pipelined


## FP Adder Hardware



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## Floating-Point Multiplication

- Consider a 4-digit decimal example
- $1.110 \times 10^{10} \times 9.200 \times 10^{-5}$

1. Add exponents

New exponent $=10+-5=5$
2. Multiply significands

$$
1.110 \times 9.200=10.212 \Rightarrow 10.212 \times 10^{5}
$$

3. Normalize result \& check for over/underflow
$1.0212 \times 10^{6}$
4. Round and renormalize if necessary $1.021 \times 10^{6}$
5. Determine sign of result from signs of operands $+1.021 \times 10^{6}$

## Floating-Point Multiplication

- Now consider a 4-digit binary example
- $1.000_{2} \times 2^{-1} \times-1.110_{2} \times 2^{-2}(0.5 \times-0.4375)$

1. Add exponents:

For biased exponents, subtract bias from sum
Unbiased: $-1+-2=-3$
Biased: $(-1+127)+(-2+127)=-3+254-127=-3+127$
2. Multiply significands
$1.000_{2} \times 1.110_{2}=1.1102 \Rightarrow 1.110_{2} \times 2^{-3}$
3. Normalize result \& check for over/underflow
$1.110_{2} \times 2^{-3}$ (no change) with no over/underflow
4. Round and renormalize if necessary
$1.110_{2} \times 2^{-3}$ (no change)
5. Determine sign: +ve $\times-\mathrm{ve} \Rightarrow-\mathrm{ve}$
$-1.110_{2} \times 2^{-3}=-0.21875$

## FP Arithmetic Hardware

- FP multiplier is of similar complexity to FP adder
- But uses a multiplier for significands instead of an adder
- FP arithmetic hardware usually does
- Addition, subtraction, multiplication, division, reciprocal, square-root
- FP $\leftrightarrow$ integer conversion
- Operations usually takes several cycles
- Can be pipelined, especially Big ALU


## FP Instructions in MIPS

- FP hardware is coprocessor 1 (c1)
- Adjunct processor that extends the ISA
- Coprocessors are alternate execution units, with register files separate from the CPU
- MIPS architecture level defines up to 4 coprocessor units, numbered 0 to 3
- Separate FP registers (FPR), cf. GPR
- 32 single-precision: \$f0, \$f1, ... \$f31
- Paired for double-precision: \$f0/\$f1, \$f2/\$f3, ...

Release 2 of MIPS ISA supports $32 \times 64$-bit FP reg's

- FP instructions operate only on FP registers
- Programs generally don't do integer ops on FP data, or vice versa
- More registers with minimal code-size impact


## FP Instructions in MIPS

- FP load and store instructions
- 1wc1, swc1; 1dc1, sdc1

$$
\text { e.g., 1dc1 \$f8, } 32(\$ s p)
$$

- Single-precision arithmetic
- add.s, sub.s, mul.s, div.s
e.g., add.s \$f0, \$f1, \$f6
- Double-precision arithmetic
- add.d, sub.d, mul.d, div.d
e.g., mul.d \$f4, \$f4, \$f6
- Single- and double-precision comparison (cf. ARM)
- c. $x x . \mathrm{s}, \mathrm{c} . x x . \mathrm{d}(x x$ is eq, $7 \mathrm{t}, 7 \mathrm{e}, \ldots$ )
- Sets or clears FP condition-code bit
e.g.c.7t.s \$f3, \$f4
- Branch on FP condition code true or false
- bc1t, bc1f
e.g., bc1t TargetLabe1


## FP Example: ${ }^{\circ} \mathrm{F}$ to ${ }^{\circ} \mathrm{C}$

- C code:
float f2c (float fahr) \{ return ((5.0/9.0)*(fahr - 32.0));
\}
- fahr in \$f12, result in \$f0, literals in global memory space
- Compiled MIPS code:
f2c: 1wc1 \$f16, const5(\$gp)
1wc2 \$f18, const9 (\$gp)
div.s \$f16, \$f16, \$f18

1wc1 \$f18, const32(\$gp)
sub.s \$f18, \$f12, \$f18
mul.s \$f0, \$f16, \$f18
jr \$ra

## FP Example: Array Multiplication

- $\mathrm{X}=\mathrm{X}+\mathrm{Y} \times \mathrm{Z}$
- All $32 \times 32$ matrices, 64-bit double-precision elements
- C code:
void mm (double x[][], double y[][], double z[][]) \{
int i, j, k;
for ( $\mathbf{i}=0 ; \mathrm{i}!=32 ; \mathbf{i}=\mathbf{i}+1$ )
for ( $j=0 ; j!=32 ; j=j+1$ )
for ( $k=0 ; k!=32 ; k=k+1$ )
$x[i][j]=x[i][j]$

$$
+y[\mathrm{i}][\mathrm{k}] * z[k][j] ;
$$

\}

- Addresses of $x, y, z$ in \$a0, \$a1, \$a2, and i, j, k in \$s0, \$s1, \$s2


## FP Example: Array Multiplication

## MIPS code:



## FP Example: Array Multiplication



## Accurate Arithmetic

- IEEE Std. 754 specifies additional rounding control
- Extra bits of precision (guard, round, sticky)
- Choice of rounding modes
- Allows programmer to fine-tune numerical behavior of a computation
- Not all FP units implement all options
- Most programming languages and FP libraries just use defaults
- Trade-off between hardware complexity, performance, and market requirements
- Fused multiply add
- A floating-point instruction that performs both a multiply and an add, but rounds only once
- A single rounding step increases the precision of multiply add
- 2 flops per instruction
- E.g madd.d fd, fr, fs, ft (MIPS64)


## Interpretation of Data

- Bits have no inherent meaning
- Interpretation depends on the instructions applied
- Computer representations of numbers
- Finite range and precision
- Need to account for this in programs


## Associativity

- Parallel programs may interleave operations in unexpected orders
- Assumptions of associativity may fail

|  |  | $(x+y)+z$ | $x+(y+z)$ |
| ---: | ---: | ---: | ---: |
| $x$ | $-1.50 E+38$ |  | $-1.50 E+38$ |
| $y$ | $1.50 E+38$ | $0.00 E+00$ |  |
| $z z$ | 1.0 | 1.0 | $1.50 E+38$ |
|  |  | $1.00 E+00$ | $0.00 E+00$ |

- Integer addition is associative, while floatingpoint addition is not associative
- Need to validate parallel programs under varying degrees of parallelism


## x86 FP Architecture

- Originally based on 8087 FP coprocessor
- $8 \times 80$-bit extended-precision registers
- Used as a push-down stack
- loads push numbers onto the stack, operations find operands in the 2 top elements of the stack, stores pop elements off the stack
- Registers indexed from TOS (Top of Stack): ST(0), ST(1), ...
- FP values are 32-bit or 64 in memory
- Converted on load/store of memory operand
- Integer operands can also be converted on load/store
- Very difficult to generate and optimize code
- Result: poor FP performance
- Intel created a more traditional fp architecture as part of SSE2


## Streaming SIMD Extension 2 (SSE2)

- Adds $8 \times 64$-bit registers
- Complier can choose to use 8 SSE FPRs
- Extended to 16 registers in AMD64/EM64T
- Can be used for multiple FP operands
- $2 \times 64$-bit double precision
- $4 \times 32$-bit double precision
- Instructions operate on them simultaneously Single-Instruction Multiple-Data


## Fallacies

- Fallacy: Just as a left shift instruction can replace an integer multiply by a power of 2 , a right shift is the same as an integer division by a power of 2
- Left shift by $i$ places multiplies an integer by $2^{i}$, e.g. s11
- Right shift divides by 2i? e.g. sr1
- Only for unsigned integers
- For signed integers
- Arithmetic right shift: replicate the sign bit, sra

$$
\text { E.g., }-5 / 4,11111011_{2} \gg 2=11111110_{2}
$$

## Pitfalls

- Pitfall: The MIPS instruction add immediate unsigned (addiu) sign-extends its 16-bit immediate field


## Fallacies

- Fallacy: Only theoretical mathematicians care about floating-point accuracy
- Important for scientific code
- But for everyday consumer use?
- "My bank balance is out by $0.0002 \phi!$ "
- The Intel Pentium FDIV bug
- In Sept. 1994, a math professor discovered the bug
- The market expects accuracy
- This recall cost Intel $\$ 500$ million!
- In April 1997, another FP bug was revealed in Pentium Pro and Pentium II
- Public acknowledgement
- Software patch


## Concluding Remarks

- ISAs support arithmetic
- Signed and unsigned integers
- Operand: 2's complement
- Floating-point approximation to reals
- Bounded range and precision
- Operations can overflow and underflow
- MIPS ISA
- Core instructions: 54 most frequently used $100 \%$ of SPECINT, $97 \%$ of SPECFP
- Other instructions: less frequent, 3\%

MIPS assembly language

| Category | Instruction |  | Example | Meaning | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Arithmetic | add | add | \$ 1 1. \$ 2. \$ ${ }^{\text {S }}$ | \$s1 = \$s 2 + \$ 3 | Three operands; overflow detected |
|  | subtract | sub | \$ 1, \$s2,\$s3 | \$s1 = \$s2-\$s3 | Three operands; overflow detected |
|  | add immediate | addi | \$ 1, \$ 2 2,100 | \$ ${ }^{\text {c }}$ = \$s2 + 100 | + constant; overflow detected |
|  | add unsigned | addu | \$ 1, \$ 2. \$ 3 | \$s1 = \$ $22+\$ \mathrm{~s} 3$ | Three operands; overflow undetected |
|  | subtract unsigned | subu | \$ 1 1,\$s2,\$s3 | \$s1 = \$s2-\$s3 | Three operands; overflow undetected |
|  | add immediate unsigned | addiu | \$s1,\$s2,100 | \$ $\mathrm{s}^{\text {c }}$ = \$ $\mathrm{s} 2+100$ | + constant; overflow undetected |
|  | move from coprocessor register | mfcO | \$s1, \$epc | \$s1 = \$epc | Copy Exception PC + special regs |
|  | multiply | muTt | \$ 2. \$ s 3 | Hi, Lo $=$ \$ S $2 \times \$$ S 3 | 64-bit signed product in Hi, Lo |
|  | multiply unsigned | multu | \$ ${ }^{\text {2 } 2 . \$ \text { s }}$ | $\mathrm{Hi}, \mathrm{Lo}=\$ \mathrm{~s} 2 \times \$ \mathrm{~s} 3$ | 64-bit unsigned product in Hi, Lo |
|  | divide | div | \$s2,\$s3 | $\begin{aligned} & \mathrm{Lo}=\$ \mathrm{~s} 2 / \$ \mathrm{~s} 3, \\ & \mathrm{Hi}=\$ \mathrm{~s} 2 \mathrm{mod} \$ \mathrm{~s} 3 \end{aligned}$ | Lo = quotient, $\mathrm{Hi}=$ remainder |
|  | divide unsigned | divu | \$s2.\$s3 | $\begin{aligned} & \mathrm{Lo}=\$ \mathrm{~s} 2 / \$ \mathrm{~s} 3, \\ & \mathrm{Hi}=\$ \mathrm{~s} 2 \mathrm{mod} \$ \mathrm{~s} 3 \end{aligned}$ | Unsigned quotient and remainder |
|  | move from Hi | mfhi | \$ 51 | \$ $\mathrm{S1}=\mathrm{Hi}$ | Used to get copy of Hi |
|  | move from Lo | mflo | \$ 51 | \$ S 1 = Lo | Used to get copy of Lo |
| Data transfer | load word | 1w | \$ 1 1,20(\$s2) | \$s1 = Memory[\$s2 + 20] | Word from memory to register |
|  | store word | SW | \$ $11,20(\$ 52)$ | Memory[\$ $22+20]=\$$ 1 1 | Word from register to memory |
|  | load half unsigned | 1hu | \$ $51.20(\$ 52)$ | \$ 12 = Memory[\$ $22+20]$ | Halfword memory to register |
|  | store half | sh | \$ $51.20(\$ 82)$ | Memory[\$s2 + 20] = \$ 11 | Halfword register to memory |
|  | load byte unsigned | 1 bu | \$s1,20(\$s2) | \$ 11 = Memory[\$ $22+20]$ | Byte from memory to register |
|  | store byte | sb | \$ $1.20(\$ 52)$ | Memory[\$s2 + 20] = \$ 11 | Byte from register to memory |
|  | load linked word | 11 | \$ $1,20(\$ 52)$ | \$s1 = Memory[\$s2 + 20] | Load word as 1st half of atomic swap |
|  | store conditional word | SC | \$s1,20(\$s2) | $\begin{aligned} & \text { Memory }[\$ \text { s } 2+20]=\$ s 1 ; \$ \text { s } 1=0 \\ & \text { or } 1 \end{aligned}$ | Store word as 2nd half atomic swap |
|  | load upper immediate | 1ui | \$51,100 | \$ $51=100 * 2^{16}$ | Loads constant in upper 16 bits |
| Logical | AND | AND | \$s1,\$s2,\$s3 | \$s1 = \$52 \& \$ 3 | Three reg. operands; bit-by-bit AND |
|  | OR | OR | \$ $11, \$$ 2, \$s3 | \$s1 = \$ $22 \mid \$ 53$ | Three reg. operands; bit-by-bit OR |
|  | NOR | NOR | \$ $51, \$ \mathrm{~s} 2, \$ \mathrm{~s} 3$ | \$s1 $=\sim(\$ 52 \mid \$ 53)$ | Three reg. operands; bit-by-bit NOR |
|  | AND immediate | ANDi | \$ $11, \$$ 2,100 | \$s1 = \$s2 \& 100 | Bit-by-bit AND with constant |
|  | OR immediate | ORi | \$ 1 1, \$ 2.100 | \$s1 = \$s2 \| 100 | Bit-by-bit OR with constant |
|  | shift left logical | s 71 | \$s1,\$s2.10 | \$S1 = \$ $22<10$ | Shift left by constant |
|  | shift right logical | srl | \$ $51 . \$$ S2,10 | \$S1 = \$ $22>10$ | Shift right by constant |
| Conditional branch | branch on equal | beq | \$51,\$s2,25 | if (\$ s1 == \$s2) go to PC + 4 + 100 | Equal test; PC-relative branch |
|  | branch on not equal | bne | \$s1,\$s2,25 | if (\$51 ! = \$s2) go to PC + 4 + 100 | Not equal test; PC-relative |
|  | set on less than | s7t | \$s1, \$s2,\$s3 | $\begin{aligned} & \text { if }(\$ s 2<\$ s 3) \$ s 1=1 \text {; } \\ & \text { else } \$ s 1=0 \end{aligned}$ | Compare less than; two's complement |
|  | set less than immediate | s7ti | \$ 1 1, \$ 2,100 | $\begin{aligned} & \text { if }(\$ s 2<100) \quad \$ s 1=1 \text {; } \\ & \text { else } \$ s 1=0 \end{aligned}$ | Compare < constant; two's complement |
|  | set less than unsigned | s7tu | \$s1,\$s2,\$s3 | $\begin{aligned} & \text { if }(\$ s 2<\$ s 3) \quad \$ s 1=1 \text {; } \\ & \text { else } \$ s 1=0 \end{aligned}$ | Compare less than; natural numbers |
|  | set less than immediate unsigned | sltiu | \$s1, \$ 2 , 100 | $\begin{aligned} & \text { if }(\$ s 2<100) \quad \$ \text { s } 1=1 \text {; } \\ & \text { else } \$ s 1=0 \end{aligned}$ | Compare < constant; natural numbers |
| Unconditional jump | jump | j | 2500 | go to 10000 | Jump to target address |
|  | jump register | jr | \$ra | go to \$ra | For switch, procedure return |
|  | jump and link | jal | 2500 | \$ra $=$ PC + 4; go to 10000 | For procedure call |

