# **Chapter 3**

#### **Arithmetic for Computers**

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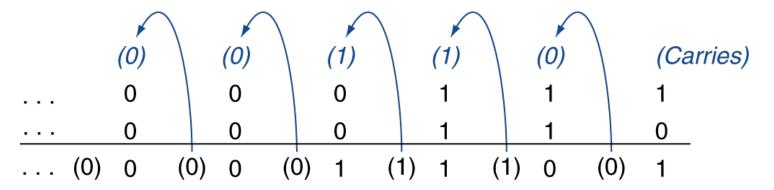
[Adapted from Computer Organization and Design, 4<sup>th</sup> Edition, Patterson & Hennessy, © 2008, MK]

### **Arithmetic for Computers**

- Operations on integers
  - Addition and subtraction
  - Multiplication and division
  - Dealing with overflow
- Floating-point real numbers
  - Representation and operations
- ALU: Arithmetic Logic Unit
  - HW performs addition, subtraction, and usually logical operations such as AND and OR

# **Integer Addition**

Example: 7 + 6



- Overflow if result out of range (2's complement)
  - Adding +ve and –ve operands, no overflow
  - Adding two +ve operands
    - Overflow if result sign is 1
  - Adding two –ve operands
    - Overflow if result sign is 0

### Integer Subtraction

- Add negation of second operand
- Example: 7 6 = 7 + (–6) +7: 0000 0000 ... 0000 0111
  - **–**6: 1111 1111 ... 1111 1010
  - +1: 0000 0000 ... 0000 0001
- Overflow if result out of range (2's complement)
  - Subtracting two +ve or two –ve operands, no overflow
  - Subtracting +ve from –ve operand
    - Overflow if result sign is 0
  - Subtracting –ve from +ve operand
    - Overflow if result sign is 1

### **Dealing with Overflow**

- Some languages ignore overflow
  - E.g., C
  - Use MIPS addu, addiu, subu instructions
  - Checked by programmer
- Other languages require raising an exception
  - E.g., Ada, Fortran
  - Use MIPS add, addi, sub instructions
  - On overflow, invoke exception handler
    - Save PC in exception program counter (EPC) register
    - Jump to predefined handler address
    - mfc0 (move from coprocessor reg) instruction can retrieve
       EPC value, to return after corrective action

## **Dealing with Overflow**

#### add rd,rs,rt

#### addu rd,rs,rt

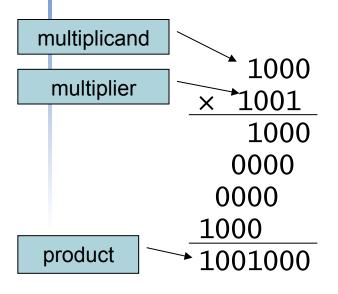
```
if (NotWordValue(GPR[rs]) or NotWordValue(GPR[rt])) then UndefinedResult() endiftemp \leftarrowGPR[rs] + GPR[rt] GPR[rd]\leftarrow sign_extend(temp<sub>31..0</sub>)
```

#### **Arithmetic for Multimedia**

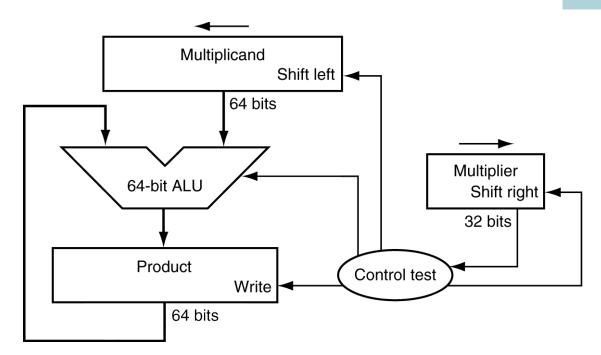
- Graphics and media processing operates on vectors of 8
   bit and 16-bit data
  - 8 bits: byte, 16 bits: halfword
  - Use 64-bit adder, with partitioned carry chain
    - Operate on 8×8-bit, 4×16-bit, or 2×32-bit vectors
  - SIMD (single-instruction, multiple-data), vector or Sub--Word Parallel
- Saturating operations
  - On overflow, result is set to largest representable value (largest positive or most negative number)
    - C.f. 2s-complement modulo arithmetic
  - E.g., clipping in audio, saturation in video
- Multimedia extensions to ISAs
  - MDMX (MIPS Digital Media eXtension), c.f. Intel SSE

# Multiplication

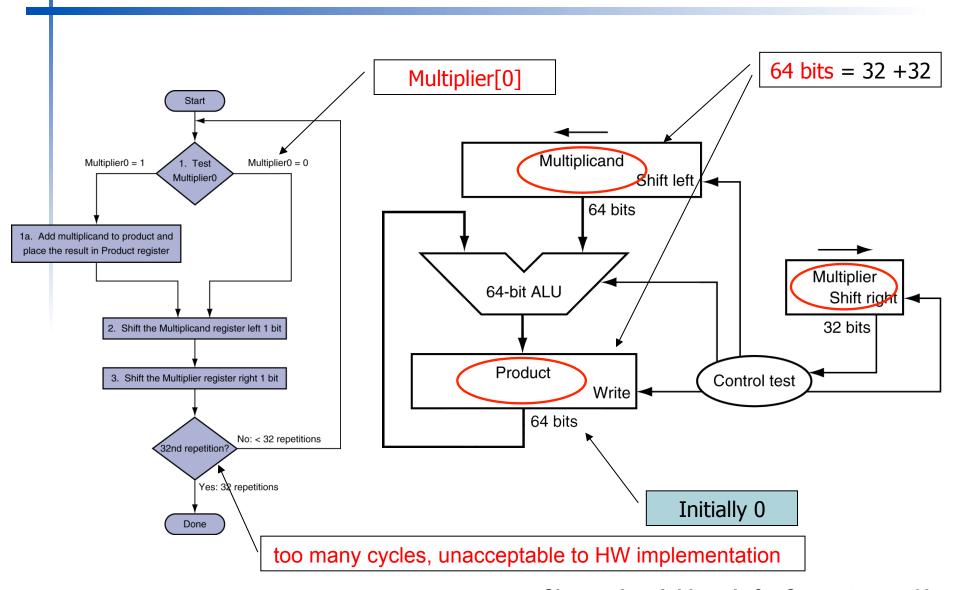
Start with long-multiplication approach



Length of product is the sum of operand lengths

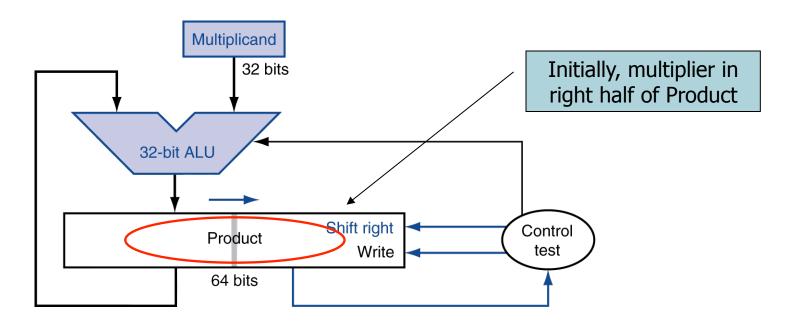


### **Multiplication Hardware**



## **Optimized Multiplier**

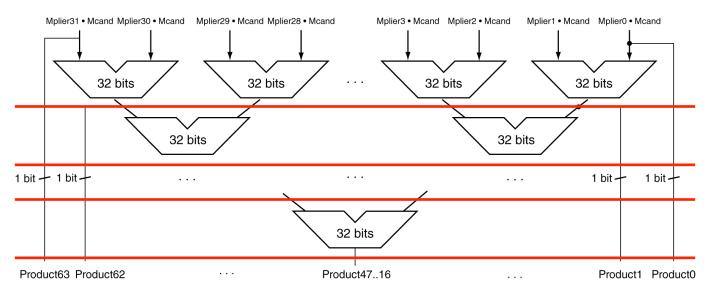
Perform steps in parallel: add/shift



- One cycle per partial-product addition
  - That's ok, if frequency of multiplications is low

## **Faster Multiplier**

- Uses multiple adders
  - Parallel tree, log<sub>2</sub>(32) or 5 32-bit add time
  - Cost/performance tradeoff



- Can be pipelined
  - Several multiplication performed in parallel
  - More resource

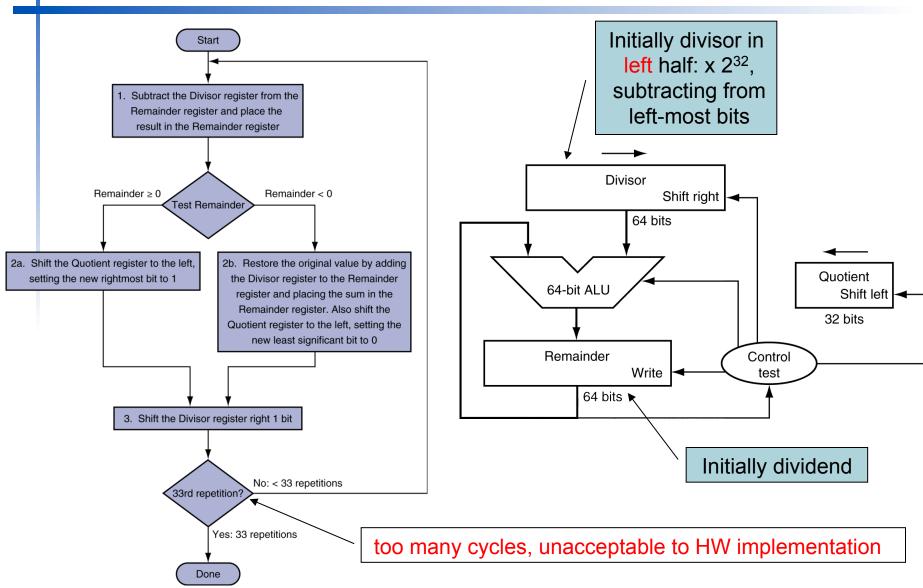
### **MIPS Integer Multiplication**

- Two special 32-bit registers for 64-bit product
  - HI: most-significant 32 bits
  - LO: least-significant 32-bits
- Instructions
  - mult rs, rt / multu rs, rt (2 operands)
    - 64-bit product in HI/LO
  - mfhi rd / mflo rd (1 operand)
    - Move from HI/LO to rd
    - Can test HI value to see if product overflows 32 bits
  - mul rd, rs, rt (32b result)
    - Least-significant 32 bits of product —> rd
    - Pseudoinstruction

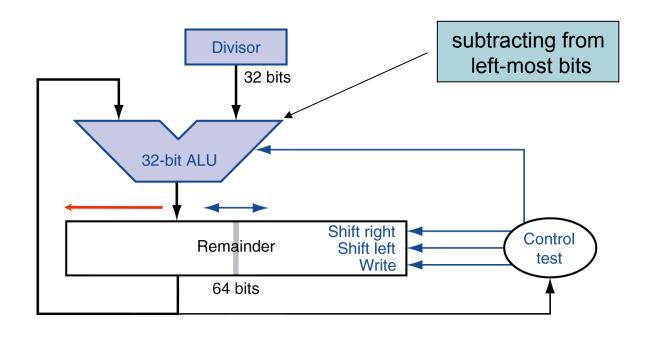
*n*-bit operands yield *n*-bit quotient and remainder

- Check for 0 divisor
- Long division approach
  - If divisor ≤ dividend bits
    - 1 bit in quotient, subtract
  - Otherwise
    - 0 bit in quotient, bring down next dividend bit
- Restoring division
  - Do the subtract, and if remainder goes < 0, add divisor back</li>
- Signed division
  - Divide using absolute values
  - Adjust sign of quotient and remainder as required

#### **Division Hardware**



#### **Optimized Divider**



- One cycle per partial-remainder subtraction
- Looks a lot like a multiplier!
  - Same hardware can be used for both

#### **Faster Division**

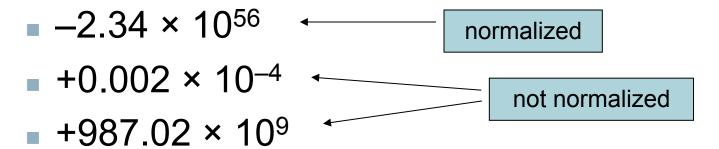
- Can't use parallel hardware as in multiplier
  - Subtraction is conditional on sign of remainder
  - As a rule, the dividend and remainder must have the same sign, no matter what the signs of the divisor and quotient
- Faster dividers (e.g. SRT division)
   generate multiple quotient bits per step
  - A typical value today is 4 bits
  - The key is guessing the value to subtract
  - Still require multiple steps

#### **MIPS Division**

- Use HI/LO registers for result as multiplication
  - HI: 32-bit remainder
  - LO: 32-bit quotient
- Instructions
  - div rs, rt / divu rs, rt
  - No overflow or divide-by-0 checking
    - Software must perform checks if required
  - Use mfhi, mflo to access result
  - div rd, rs, rt: pseudoinstruction

# Floating Point

- Representation for non-integral numbers
  - Including very small and very large numbers
- Like scientific notation



- In binary
  - $\bullet$  ±1. $xxxxxxxx_2 \times 2^{yyyy}$
- Types float and double in C

#### Floating Point Standard

- Defined by IEEE Std. 754-1985
- Developed in response to divergence of representations
  - Portability issues for scientific code
- Now almost universally adopted
- Two representations
  - Single precision (32-bit)
  - Double precision (64-bit)

# **IEEE Floating-Point Format**

single: 8 bits single: 23 bits double: 11 bits double: 52 bits

S Exponent Fraction

$$x = (-1)^{S} \times (1 + Fraction) \times 2^{(Exponent-Bias)}$$

- S: sign bit  $(0 \Rightarrow \text{non-negative}, 1 \Rightarrow \text{negative})$
- Normalize significand: 1.0 ≤ |significand| < 2.0</p>
  - Always has a leading pre-binary-point 1 bit, so no need to represent it explicitly (hidden bit)
  - Significand is Fraction with the "1." To restore: 1+Fraction
- Exponent: excess representation: actual exponent + Bias
  - Ensures exponent is unsigned, biased notation
  - IEEE 754's bias

Single: Bias = 127 =  $2^7$ -1; Double: Bias = 1203 =  $2^{10}$ -1

Bias is 2<sup>(n-1)</sup>-1, c.f. conventional 2<sup>(n-1)</sup>

### Floating-Point Example

- Represent –0.75
  - $-0.75 = (-1)^1 \times 1.1_2 \times 2^{-1}$
  - S = 1
  - Fraction =  $1000...00_2$
  - Exponent = -1 + Bias
    - Single:  $-1 + 127 = 126 = 011111110_2$
    - Double:  $-1 + 1023 = 1022 = 0111111111110_2$
- Single: 10111111101000...00
- Double: 10111111111101000....00

#### Floating-Point Example

What number is represented by the single -precision float

11000000101000...00

- S = 1
- Fraction =  $01000...00_2$
- Exponent =  $10000001_2 = 129$

$$x = (-1)^{1} \times (1 + 01_{2}) \times 2^{(129 - 127)}$$

$$= (-1) \times 1.25 \times 2^{2}$$

$$= -5.0$$

## Single-Precision Range

- Exponents 00000000 and 11111111 reserved
- Smallest value
  - Exponent: 00000001⇒ actual exponent = 1 - 127 = -126
  - Fraction:  $000...00 \Rightarrow \text{significand} = 1.0$
  - $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- Largest value
  - exponent: 11111110⇒ actual exponent = 254 127 = +127
  - Fraction: 111...11 ⇒ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$

#### **Double-Precision Range**

- Exponents 0000...00 and 1111...11 reserved
- Smallest value
  - Exponent: 0000000001⇒ actual exponent = 1 - 1023 = -1022
  - Fraction:  $000...00 \Rightarrow \text{significand} = 1.0$
  - $\pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- Largest value
  - Exponent: 11111111110⇒ actual exponent = 2046 1023 = +1023
  - Fraction: 111...11 ⇒ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$

#### Floating-Point Precision

- Relative precision
  - All fraction bits are significant
  - Single: approx 2<sup>-23</sup>
    - Equivalent to 23 × log<sub>10</sub>2 ≈ 23 × 0.3 ≈ 6 decimal digits of precision
  - Double: approx 2<sup>-52</sup>
    - Equivalent to 52 × log<sub>10</sub>2 ≈ 52 × 0.3 ≈ 16 decimal digits of precision

#### **Denormal Numbers**

Exponent = 000...0 ⇒ hidden bit is 0

$$x = (-1)^{S} \times (0 + Fraction) \times 2^{-Bias}$$

- Smaller than normal numbers
  - allow for gradual underflow, with diminishing precision
- Denormal with fraction = 000...0

$$x = (-1)^{S} \times (0+0) \times 2^{-Bias} = \pm 0.0$$
Two representations of 0.0!

#### Infinities and NaNs

- Exponent = 111...1, Fraction = 000...0
  - ±Infinity
  - Can be used in subsequent calculations, avoiding need for overflow check
- Exponent = 111...1, Fraction ≠ 000...0
  - Not-a-Number (NaN)
  - Indicates illegal or undefined result
    - e.g., 0.0 / 0.0
  - Can be used in subsequent calculations

### Floating-Point Addition

- Consider a 4-digit decimal example
  - $\bullet$  9.999 × 10<sup>1</sup> + 1.610 × 10<sup>-1</sup>
- 1. Align decimal points

Align the number with smaller exponent, match the larger exponent  $9.999 \times 10^1 + 0.016 \times 10^1$ 

2. Add significands

$$9.999 \times 10^{1} + 0.016 \times 10^{1} = 10.015 \times 10^{1}$$

- 3. Normalize result & check for over/underflow 1.0015 × 10<sup>2</sup>
- 4. Round and renormalize if necessary

$$1.002 \times 10^{2}$$

### Floating-Point Addition

- Now consider a 4-digit binary example
  - $-1.000_2 \times 2^{-1} + -1.110_2 \times 2^{-2} (0.5 + -0.4375)$
- 1. Align binary points

Shift number with smaller exponent, match the larger exponent  $1.000_2 \times 2^{-1} + -0.111_2 \times 2^{-1}$ 

2. Add significands

$$1.000_2 \times 2^{-1} + -0.111_2 \times 2^{-1} = 0.001_2 \times 2^{-1}$$

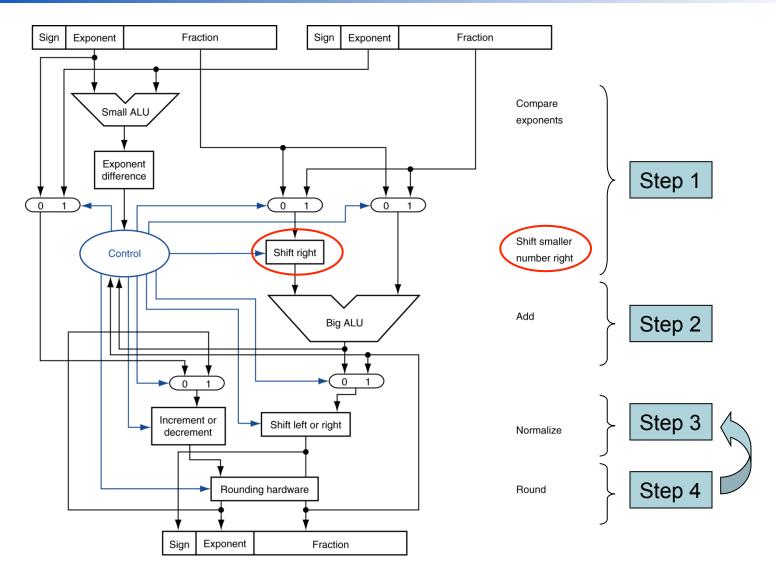
- 3. Normalize result & check for over/underflow
  - $1.000_2 \times 2^{-4}$ , with no over/underflow
- 4. Round and renormalize if necessary

$$1.000_2 \times 2^{-4}$$
 (no change) =  $0.0625$ 

#### **FP Adder Hardware**

- Much more complex than integer adder
- Doing it in one clock cycle would take too long
  - Much longer than integer operations
  - Slower clock would penalize all instructions
- FP adder usually takes several cycles
  - Can be pipelined

#### **FP Adder Hardware**



**Chapter 3 — Arithmetic for Computers — 32** 

## Floating-Point Multiplication

- Consider a 4-digit decimal example
  - $\bullet$  1.110 × 10<sup>10</sup> × 9.200 × 10<sup>-5</sup>
- 1. Add exponents

New exponent = 
$$10 + -5 = 5$$

2. Multiply significands

$$1.110 \times 9.200 = 10.212 \implies 10.212 \times 10^{5}$$

3. Normalize result & check for over/underflow

$$1.0212 \times 10^6$$

4. Round and renormalize if necessary

$$1.021 \times 10^6$$

5. Determine sign of result from signs of operands

$$+1.021 \times 10^{6}$$

# Floating-Point Multiplication

- Now consider a 4-digit binary example
  - $1.000_2 \times 2^{-1} \times -1.110_2 \times 2^{-2} (0.5 \times -0.4375)$
- 1. Add exponents:

For biased exponents, subtract bias from sum

Unbiased: -1 + -2 = -3

Biased: 
$$(-1 + 127) + (-2 + 127) = -3 + 254 - 127 = -3 + 127$$

2. Multiply significands

$$1.000_2 \times 1.110_2 = 1.1102 \implies 1.110_2 \times 2^{-3}$$

- 3. Normalize result & check for over/underflow
  - $1.110_2 \times 2^{-3}$  (no change) with no over/underflow
- 4. Round and renormalize if necessary

$$1.110_2 \times 2^{-3}$$
 (no change)

5. Determine sign: +ve  $\times$  -ve  $\Rightarrow$  -ve

$$-1.110_2 \times 2^{-3} = -0.21875$$

Biased sum !!

#### **FP Arithmetic Hardware**

- FP multiplier is of similar complexity to FP adder
  - But uses a multiplier for significands instead of an adder
- FP arithmetic hardware usually does
  - Addition, subtraction, multiplication, division, reciprocal, square-root
  - FP ⇔ integer conversion
- Operations usually takes several cycles
  - Can be pipelined, especially Big ALU

#### **FP Instructions in MIPS**

- FP hardware is coprocessor 1 (c1)
  - Adjunct processor that extends the ISA
  - Coprocessors are alternate execution units, with register files separate from the CPU
  - MIPS architecture level defines up to 4 coprocessor units, numbered 0 to 3
- Separate FP registers (FPR), cf. GPR
  - 32 single-precision: \$f0, \$f1, ... \$f31
  - Paired for double-precision: \$f0/\$f1, \$f2/\$f3, ...
    - Release 2 of MIPS ISA supports 32 × 64-bit FP reg's
- FP instructions operate only on FP registers
  - Programs generally don't do integer ops on FP data, or vice versa
  - More registers with minimal code-size impact

#### **FP Instructions in MIPS**

- FP load and store instructions
  - lwc1, swc1; ldc1, sdc1e.g., ldc1 \$f8, 32(\$sp)
- Single-precision arithmetic
  - add.s, sub.s, mul.s, div.se.g., add.s \$f0, \$f1, \$f6
- Double-precision arithmetic
  - add.d, sub.d, mul.d, div.de.g., mul.d \$f4, \$f4, \$f6
- Single- and double-precision comparison (cf. ARM)
  - c.xx.s, c.xx.d (xx is eq, lt, le, ...)
  - Sets or clears FP condition-code bit
    - e.g. c.lt.s \$f3, \$f4
- Branch on FP condition code true or false
  - bc1t, bc1f
    - e.g., bc1t TargetLabel

# FP Example: °F to °C

C code:

```
float f2c (float fahr) {
  return ((5.0/9.0)*(fahr - 32.0));
}
```

- fahr in \$f12, result in \$f0, literals in global memory space
- Compiled MIPS code:

```
f2c: lwc1  $f16, const5($gp)
  lwc2  $f18, const9($gp)
  div.s  $f16, $f16, $f18
  lwc1  $f18, const32($gp)
  sub.s  $f18, $f12, $f18
  mul.s  $f0, $f16, $f18
  jr  $ra
```

## FP Example: Array Multiplication

- $X = X + Y \times Z$ 
  - All 32 × 32 matrices, 64-bit double-precision elements
- C code:

Addresses of x, y, z in \$a0, \$a1, \$a2, and i, j, k in \$s0, \$s1, \$s2

## FP Example: Array Multiplication

#### MIPS code:

```
1i $t1, 32 # $t1 = 32 (row size/loop end)
   li $s0, 0 # i = 0; initialize 1st for loop
L1: li \$s1, 0 # j = 0; restart 2nd for loop
L2: 1i $s2, 0 # k = 0; restart 3rd for loop
   sll $t2, $s0, 5  # $t2 = i * 32  (size of row of x)
   addu t2, t2, s1 # t2 = i * size(row) + j
   sll $t2, $t2, 3 # $t2 = byte offset of [i][j]
   addu t2, a0, t2 # t2 = byte address of <math>x[i][j]
   1.d f4, 0(t2) # f4 = 8 bytes of x[i][j]
L3: s11 $t0, $s2, 5 # $t0 = k * 32 (size of row of z)
   addu t0, t0, s1 # t0 = k * size(row) + i
   sll $t0, $t0, 3 # $t0 = byte offset of [k][j]
   addu t0, a2, t0 # t0 = byte address of <math>z[k][j]
   1.d f16, 0(t0) # f16 = 8 bytes of z[k][j]
```

•••

## FP Example: Array Multiplication

```
\$11 \$t0, \$s0, 5  # \$t0 = i*32 (size of row of y)
addu t0, t0, s2 # t0 = i*size(row) + k
sll $t0, $t0, 3 # $t0 = byte offset of [i][k]
addu t0, a1, t0 # t0 = byte address of y[i][k]
1.d f18, 0(t0) # f18 = 8 bytes of y[i][k]
mul.d f16, f18, f16 # f16 = y[i][k] * z[k][j]
add.d $f4, $f4, $f16 # f4=x[i][j] + y[i][k]*z[k][j]
addiu $s2, $s2, 1 # $k k + 1
bne \$s2, \$t1, L3 # if (k != 32) go to L3
s.d f4, O(t2) # x[i][j] = f4
addiu \$\$1, \$\$1, 1 # \$j = j + 1
bne $s1, $t1, L2 # if (j != 32) go to L2
addiu $s0, $s0, 1 # $i = i + 1
bne $s0, $t1, L1 # if (i != 32) go to L1
```

### **Accurate Arithmetic**

- IEEE Std. 754 specifies additional rounding control
  - Extra bits of precision (guard, round, sticky)
  - Choice of rounding modes
  - Allows programmer to fine-tune numerical behavior of a computation
- Not all FP units implement all options
  - Most programming languages and FP libraries just use defaults
- Trade-off between hardware complexity, performance, and market requirements
- Fused multiply add
  - A floating-point instruction that performs both a multiply and an add, but rounds only once
  - A single rounding step increases the precision of multiply add
  - 2 flops per instruction
  - E.g madd.d fd, fr, fs, ft (MIPS64)

## Interpretation of Data

#### **The BIG Picture**

- Bits have no inherent meaning
  - Interpretation depends on the instructions applied
- Computer representations of numbers
  - Finite range and precision
  - Need to account for this in programs

# **Associativity**

- Parallel programs may interleave operations in unexpected orders
  - Assumptions of associativity may fail

		(x+y)+z	x+(y+z)
X	-1.50E+38		-1.50E+38
У	1.50E+38	0.00E+00	
Z	1.0	1.0	1.50E+38
		1.00E+00	0.00E+00

- Integer addition is associative, while floatingpoint addition is not associative
- Need to validate parallel programs under varying degrees of parallelism

#### x86 FP Architecture

- Originally based on 8087 FP coprocessor
  - 8 × 80-bit extended-precision registers
  - Used as a push-down stack
    - loads push numbers onto the stack, operations find operands in the 2 top elements of the stack, stores pop elements off the stack
  - Registers indexed from TOS (Top of Stack):
    - ST(0), ST(1), ...
- FP values are 32-bit or 64 in memory
  - Converted on load/store of memory operand
  - Integer operands can also be converted on load/store
- Very difficult to generate and optimize code
  - Result: poor FP performance
  - Intel created a more traditional fp architecture as part of SSE2

## **Streaming SIMD Extension 2 (SSE2)**

- Adds 8 × 64-bit registers
  - Complier can choose to use 8 SSE FPRs
  - Extended to 16 registers in AMD64/EM64T
- Can be used for multiple FP operands
  - 2 × 64-bit double precision
  - 4 × 32-bit double precision
  - Instructions operate on them simultaneously
    - Single-Instruction Multiple-Data

#### **Fallacies**

- Fallacy: Just as a left shift instruction can replace an integer multiply by a power of 2, a right shift is the same as an integer division by a power of 2
  - Left shift by i places multiplies an integer by 2<sup>i</sup>, e.g. s11
  - Right shift divides by 2<sup>i</sup>? e.g. sr<sup>1</sup>
    - Only for unsigned integers
  - For signed integers
    - Arithmetic right shift: replicate the sign bit, Sra
    - E.g., -5/4,  $11111011_2 >> 2 = 111111110_2$

#### **Pitfalls**

 Pitfall: The MIPS instruction add immediate unsigned (addiu) sign-extends its 16-bit immediate field

#### **Fallacies**

- Fallacy: Only theoretical mathematicians care about floating-point accuracy
  - Important for scientific code
    - But for everyday consumer use?
      - "My bank balance is out by 0.0002¢!"
  - The Intel Pentium FDIV bug
    - In Sept. 1994, a math professor discovered the bug
    - The market expects accuracy
    - This recall cost Intel \$500 million!
  - In April 1997, another FP bug was revealed in Pentium Pro and Pentium II
    - Public acknowledgement
  - Software patch

# **Concluding Remarks**

- ISAs support arithmetic
  - Signed and unsigned integers
  - Operand: 2's complement
  - Floating-point approximation to reals
- Bounded range and precision
  - Operations can overflow and underflow
- MIPS ISA
  - Core instructions: 54 most frequently used
    - 100% of SPECINT, 97% of SPECFP
  - Other instructions: less frequent, 3%

#### MIPS assembly language

Category	Instruction	E	xample	Meaning	Comments
Arithmetic	add	add	\$s1,\$s2,\$s3	\$s1 = \$s2 + \$s3	Three operands; overflow detected
	subtract	sub	\$s1,\$s2,\$s3	\$s1 = \$s2 - \$s3	Three operands; overflow detected
	add immediate	addi	\$s1,\$s2,100	\$s1 = \$s2 + <b>100</b>	+ constant; overflow detected
	add unsigned	addu	\$s1,\$s2,\$s3	\$s1 = \$s2 + \$s3	Three operands; overflow undetected
	subtract unsigned	subu	\$s1,\$s2,\$s3	\$s1 = \$s2 - \$s3	Three operands; overflow undetected
	add immediate unsigned	addiu	\$s1,\$s2,100	\$s1 = \$s2 + <b>100</b>	+ constant; overflow undetected
	move from coprocessor register	mfc0	\$s1,\$epc	\$s1 = \$epc	Copy Exception PC + special regs
	multiply	mult	\$\$2,\$\$3	Hi, Lo = \$s2 × \$s3	64-bit signed product in Hi, Lo
	multiply unsigned	multu	<b>\$</b> s2, <b>\$</b> s3	Hi, Lo = $$s2 \times $s3$	64-bit unsigned product in Hi, Lo
	divide	div	\$s2,\$s3	Lo = \$s2 / \$s3, Hi = \$s2 mod \$s3	Lo = quotient, Hi = remainder
	divide unsigned	divu	\$s2,\$s3	Lo = \$s2 / \$s3, Hi = \$s2 mod \$s3	Unsigned quotient and remainder
	move from Hi	mfhi	<b>\$</b> s1	\$s1 = <b>Hi</b>	Used to get copy of Hi
	move from Lo	mflo	<b>\$</b> s1	\$s1 = Lo	Used to get copy of Lo
Data transfer	load word	1 w	\$s1,20(\$s2)	\$s1 = Memory[\$s2 + 20]	Word from memory to register
	store word	SW	\$s1,20(\$s2)	Memory[\$s2 + 20] = \$s1	Word from register to memory
	load half unsigned	1hu	\$s1,20(\$s2)	\$s1 = Memory[\$s2 + 20]	Halfword memory to register
	store half	sh	\$s1,20(\$s2)	Memory[\$s2 + 20] = \$s1	Halfword register to memory
	load byte unsigned	1bu	\$s1,20(\$s2)	\$s1 = Memory[\$s2 + 20]	Byte from memory to register
	store byte	sb	\$s1,20(\$s2)	Memory[\$s2 + 20] = \$s1	Byte from register to memory
	load linked word	11	\$s1,20(\$s2)	\$s1 = Memory[\$s2 + 20]	Load word as 1st half of atomic swa
	store conditional word	sc	\$s1,20(\$s2)	Memory[\$s2+20]=\$s1;\$s1=0 or 1	Store word as 2nd half atomic swap
	load upper immediate	lui	\$s1,100	\$s1 = 100 * 2 <sup>16</sup>	Loads constant in upper 16 bits
Logical	AND	AND	\$s1,\$s2,\$s3	\$s1 = \$s2 & \$s3	Three reg. operands; bit-by-bit AND
	OR	OR	\$s1,\$s2,\$s3	\$s1 = \$s2   \$s3	Three reg. operands; bit-by-bit OR
	NOR	NOR	\$s1,\$s2,\$s3	\$s1 = ~ (\$s2  \$s3)	Three reg. operands; bit-by-bit NOR
	AND immediate	ANDi	\$s1,\$s2,100	\$s1 = \$s2 & 100	Bit-by-bit AND with constant
	OR immediate	ORi	\$s1,\$s2,100	\$s1 = \$s2   100	Bit-by-bit OR with constant
	shift left logical	s11	\$s1,\$s2,10	\$\$1 = \$\$2 << 10	Shift left by constant
	shift right logical	srl	\$s1,\$s2,10	\$s1 = \$s2 >> 10	Shift right by constant
	branch on equal	beg	\$s1,\$s2,25	if ( $\$\$1 == \$\$2$ ) go to PC + 4 + 100	Equal test; PC-relative branch
Condi- tional branch	branch on not equal	bne	\$s1,\$s2,25	if (\$s1!= \$s2) go to PC + 4 + 100	Not equal test; PC-relative
	set on less than	slt	\$\$1,\$\$2,\$\$3	if (\$s2 < \$s3) \$s1 = 1; else \$s1 = 0	Compare less than; two's complement
	set less than immediate	slti	\$s1,\$s2,100	if (\$s2 < 100) \$s1 = 1; else \$s1=0	Compare < constant; two's complement
	set less than unsigned	sltu	\$s1,\$s2,\$s3	if (\$s2 < \$s3) \$s1 = 1; else \$s1=0	Compare less than; natural numbers
	set less than immediate unsigned	sltiu	\$s1,\$s2,100	if (\$s2 < 100) \$s1 = 1; else \$s1 = 0	Compare < constant; natural numbers
Uncondi- tional jump	jump	j	2500	go to 10000	Jump to target address
	jump register	jr	\$ra	go to \$ra	For switch, procedure return
	jump and link	jal	2500	\$ra = PC + 4; go to 10000	For procedure call

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